Static Analysis

COMP 520: Compiler Design (4 credits)

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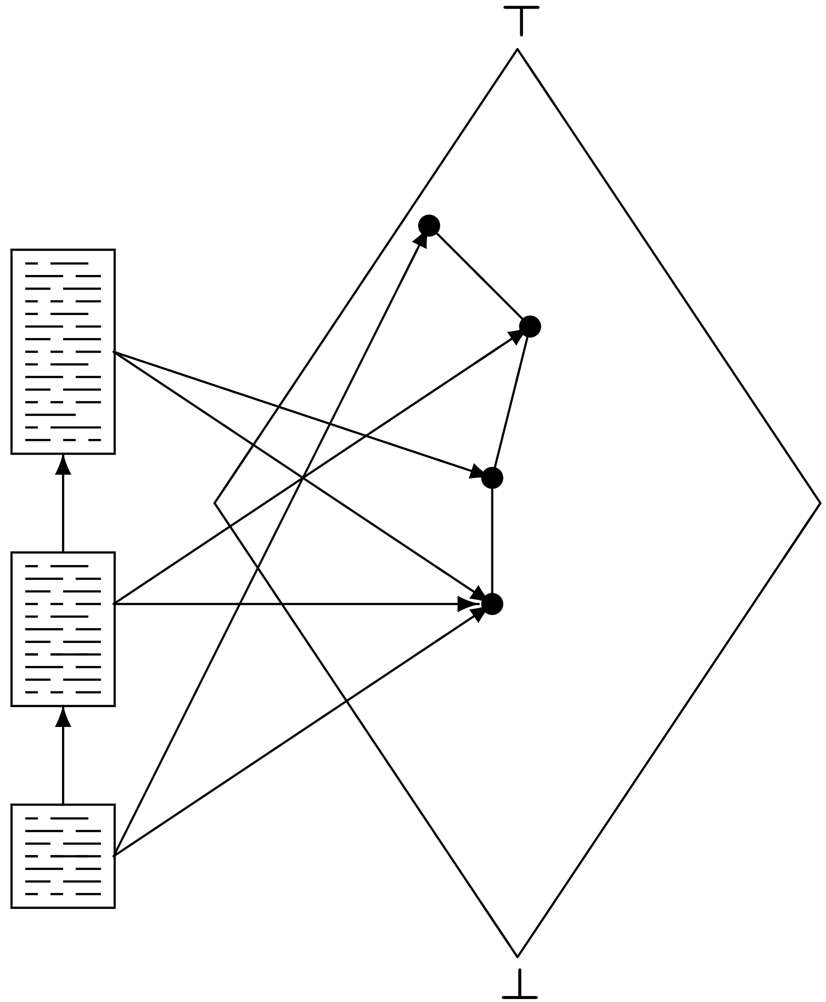


Static analysis determines interesting properties of programs to enable some optimizations.

All interesting properties are actually undecidable, so the analysis computes a conservative approximation:

- if we say *yes*, then the property definitely holds;
- if we say *no*, then the property may or may not hold;
- only the *yes* answer will help us to perform the optimization;
- a trivial analysis will say *no* always; so
- the art is to say *yes* as often as possible.

Properties need not be simply *yes* or *no*, in which case the notion of *approximation* is more subtle.

Static analysis may take place:

- at the source code level;
- at some intermediate level; or
- at the machine code level.

Static analysis may look at:

- basic blocks only (called *local analysis*);
- an entire function (called *intraprocedural* or *global analysis*); or
- the whole program (called *whole program* or *interprocedural analysis*).

In each case, we are maximally pessimistic at the boundaries.

The precision and cost of an analysis rises as we include more information.

Simple static analysis:

- is merely advanced weeding;
- uses symbol and type information; and
- is recursive in the program syntax.

An example is the *definite assignment* requirement in Java and JOOS:

- local variables must be assigned before they are read;
- this is undecidable; but
- the language specification dictates a specific conservative approximation.

For each program point, compute a set of local variables that:

- contains only variables that have definitely been assigned;
- may be too small, since the analysis is conservative; and
- depends on the set computed for the previous program point.

It accepts:

```
{ int k;
  if (flag) k = 3; else k = 4;
  System.out.println(k);
}
```

but rejects:

```
{ int k;
  if (flag) k = 3;
  if (!flag) k = 4;
  System.out.println(k);
}
```

JOOS code for statements:

```
ASNSET *defasnSTATEMENT(STATEMENT *s, ASNSET *before)
{ if (s!=NULL) {
    switch (s->kind) {
      case skipK:
          return before;
      case expK:
          return defasnEXP(s->val.expS,before);
      case returnK:
          if (s->val.returnS!=NULL)
            (void)defasnEXP(s->val.returnS,before);
          return setUniversal();
      case sequenceK:
          return
           defasnSTATEMENT(s->val.sequenceS.second,
             defasnSTATEMENT(s->val.sequenceS.first,
                             before)
           );
     ...
```

... continued

```
ASNSET *defasnSTATEMENT(STATEMENT *s, ASNSET *before)
  { ...
      case ifelseK:
          return
            setIntersect(
              defasnSTATEMENT(s->val.ifelseS.thenpart,
                defasnEXPassume(s->val.ifelseS.condition,
                                before,1)
              ),
              defasnSTATEMENT(s->val.ifelseS.elsepart,
                defasnEXPassume(s->val.ifelseS.condition,
                                before,0)
              )
            );
      case ...
  } }
```

To make the analysis more precise, it considers boolean expressions in more detail.

The procedure `defasnEXPassume(...,b)` assumes the expression evalutes to `b`.

This refinement handles a case like:

```
{ int k;
  if (a>0 && (k=b)>0)
    System.out.println(k);
}
```

which would otherwise be rejected.

In general, a static analysis becomes more precise when it may make further assumptions about the context.

The definite assignment analysis is particularly simple:

there are no recursive dependencies between the computed sets.

This allows a simple implementation:
a top-down traversal of the parse tree.

For more sophisticated analyses, we generate equations and compute the solution as a *fixed point*.

For the JIT compiler, we want to optimize the use of registers:

```
mov 1,R3        ⟹     mov 1,R1
mov R3,R1
```

This requires knowledge about the future uses of registers:

The optimization is only sound if the value of `R3` is not used later on.

For basic block register allocation, which variables need to be written back to memory?

The naïve scheme:

- must write all those variables that are *only* in registers.

A better scheme:

- write all those variables that are only in registers *and* whose values might be used later on.

This could avoid many useless spills.

In both examples, we need to know if some R_i might be used later on. If so, it is called *live*; otherwise, it is called *dead*.

A static analysis can conservatively approximate liveness at each program point.

Exact liveness is of course undecidable.

A trivial analysis will call everything live, which precludes all optimizations.

A superior analysis will identify more dead variables.

Liveness analysis for VirtualRISC:

- build a control flow graph (goto graph);
- define dataflow equations for each node;
- compute the least solution of these equations.

For basic blocks the computation is trivial.

For intraprocedural analysis we must compute a minimal fixed point in a lattice.

Consider a simple basic block:

```
mov 3,R1
mov 4,R2
add R1,R2,R3
mov R3,R0
return
```

The underlined registers are written (defined), the others are merely read (used).

The control flow graph is:

```
S1:  mov 3,R1
      |
      v
S2:  mov 4,R2
      |
      v
S3:  add R1,R2,R3
      |
      v
S4:  mov R3,R0
      |
      v
S5:  return
```

Each instruction uses some registers and defines some registers:

		uses(S_i)	defines(S_i)
S1:	mov 3,R1	{}	{R1}
S2:	mov 4,R2	{}	{R2}
S3:	add R1,R2,R3	{R1,R2}	{R3}
S4:	mov R3,R0	{R3}	{R0}
S5:	return	{R0}	{}

The register R0 is implicitly used for the return value.

Let out(S_i) be the variables that are live just *after* S_i and in(S_i) those that are live just *before* S_i:

↓ in(S_i)

S_i: `op X,Y,Z`

↓ out(S_i)

Then we have the dataflow equation:

in(S_i) = uses(S_i) $\cup$ (out(S_i) — defines(S_i))

We add those registers that are used in the current instruction and delete those that are defined here.

Since out(`S5`) = `{ }`, it follows that:

in(`S5`) = uses(`S5`) = `{R0}`

We can continue to unravel the equations:

out(`S4`) = in(`S5`) = `{R0}`
in(`S4`) = uses(`S4`) ∪ (out(`S4`) — defines(`S4`))
= `{R3}` ∪ (`{R0}` — `{R0}`)
= `{R3}`
out(`S3`) = in(`S4`) = `{R3}`
in(`S3`) = uses(`S3`) ∪ (out(`S3`) — defines(`S3`))
= `{R1,R2}` ∪ (`{R3}` — `{R3}`)
= `{R1,R2}`

and so on:

		uses(S_i)	defines(S_i)	in(S_i)
S1:	`mov 3,R1`	{}	{R1}	{}
S2:	`mov 4,R2`	{}	{R2}	{R1}
S3:	`add R1,R2,R3`	{R1,R2}	{R3}	{R1,R2}
S4:	`mov R3,R0`	{R3}	{R0}	{R3}
S5:	`return`	{R0}	{}	{R0}

In basic blocks we use the equation:

$$\text{out}(\mathbb{S}_i) = \text{in}(\mathbb{S}_{i+1})$$

If we have branches, then a node in the control flow graph may have several successors.

In this case, we must use the equation:

$$\text{out}(\mathbb{S}_i) = \bigcup_{x \,\in\, \text{succ}(\mathbb{S}_i)} \text{in}(x)$$

But now the equations are cyclic and cannot simply be unraveled.

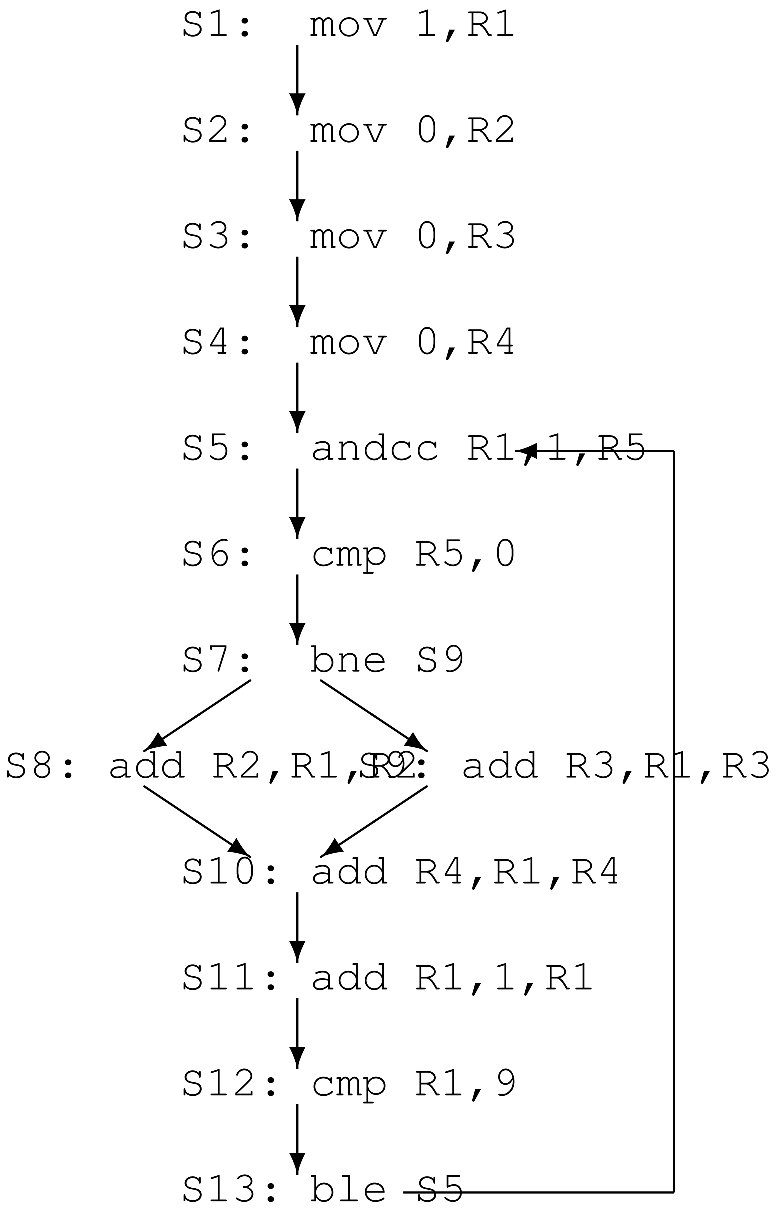
Consider the small piece of C code:

```
{ int i, sum_even, sum_odd, sum;
  i = 1;
  sum_even = 0;
  sum_odd = 0;
  sum = 0;
  while (i < 10)
   { if (i%2 == 0) sum_even = sum_even + i;
     else sum_odd = sum_odd + i;
     sum = sum + i;
     i++;
   }
}
```

It yields the following VirtualRISC code:

```
        mov 1,R1            // R1 is i
        mov 0,R2            // R2 is sum_even
        mov 0,R3            // R3 is sum_odd
        mov 0,R4            // R4 is sum
loop:
        andcc R1,1,R5      // R5 = R1 & 1
        cmp R5,0
        bne else           // if R5 != 0 goto else
        add R2,R1,R2       // R2 = R2 + R1; even case
        b endif
else:
        add R3,R1,R3       // R3 = R3 + R1; odd case
endif:
        add R4,R1,R4       // R4 = R4 + R1; update sum
        add R1,1,R1        // R1 = R1 + 1;  increment i
        cmp R1,9
        ble loop           // if i <= 9 goto loop
```

The control flow graph:



To unravel the liveness equations, we should start with:

$$\text{out}(\texttt{S13}) = \text{in}(\texttt{S5})$$

... but we have not computed in($\texttt{S5}$) yet, so this will not work!

If in($\texttt{S1}$),...,in($\texttt{S13}$) are known, then we can unravel the code as before and obtain the sets in($\texttt{S1}$),...,in($\texttt{S13}$) once again.

But this means that unraveling is a function:

$$\boldsymbol{f : \mathcal{P}(R)^{13} \rightarrow \mathcal{P}(R)^{13}}$$

where $\boldsymbol{R = \{\texttt{R1}, \texttt{R2}, \ldots, \texttt{R5}\}}$.

A solution is a fixed point, and we want the minimal one.

Two fundamental observations:

- the set $D = \mathcal{P}(R)^{13}$ is a finite *lattice*:

$$\forall x, y \in D : x \sqcap y \in D \ \wedge \ x \sqcup y \in D$$

  where $\sqsubseteq$ is point-wise set inclusion; and

- the unraveling function f is *monotonic*:

$$\forall x, y \in D : x \sqsubseteq y \Rightarrow f(x) \sqsubseteq f(y)$$

  since $g(x) = A \cup (x - B)$ is monotonic.

The fixed point theorem:

Any monotonic function f on a finite lattice D has the unique minimal fixed point:

$$\bigsqcup_i f^i(\bot)$$

which is always obtained after finitely many iterations.

For $D = \mathcal{P}(R)^{13}$ we have that:

$$\perp = (\emptyset, \emptyset, \ldots, \emptyset)$$

so we start with the sets in(S_i) = { } and keep unraveling until they no longer change.

Note that:

$$\top = (R, R, \ldots, R)$$

is always a safe answer, but clearly useless and pessimistic.

Observe that the maximal fixed-point:

$$\sqcap_i f^i(\top)$$

may in general be smaller than $\top$.

Computing the minimal fixed point:

	uses	defs	succ	$\perp$	$f(\perp)$	$f^2(\perp)$
S1		R1	S2	{}	{}	{}
S2		R2	S3	{}	{}	{}
S3		R3	S4	{}	{}	{}
S4		R4	S5	{}	{}	{R1}
S5	R1	R5	S6	{}	{R1}	{R1}
S6	R5		S7	{}	{R5}	{R5}
S7			S8,S9	{}	{}	{R1,R2,R3}
S8	R1,R2	R2	S10	{}	{R1,R2}	{R1,R2,R4}
S9	R1,R3	R3	S10	{}	{R1,R3}	{R1,R3,R4}
S10	R1,R4	R4	S11	{}	{R1,R4}	{R1,R4}
S11	R1	R1	S12	{}	{R1}	{R1}
S12	R1		S13	{}	{R1}	{R1}
S13			S5	{}	{}	{R1}

The function is:

$$f(X_1, X_2, \ldots, X_{13}) = (Y_1, Y_2, \ldots, Y_{13})$$

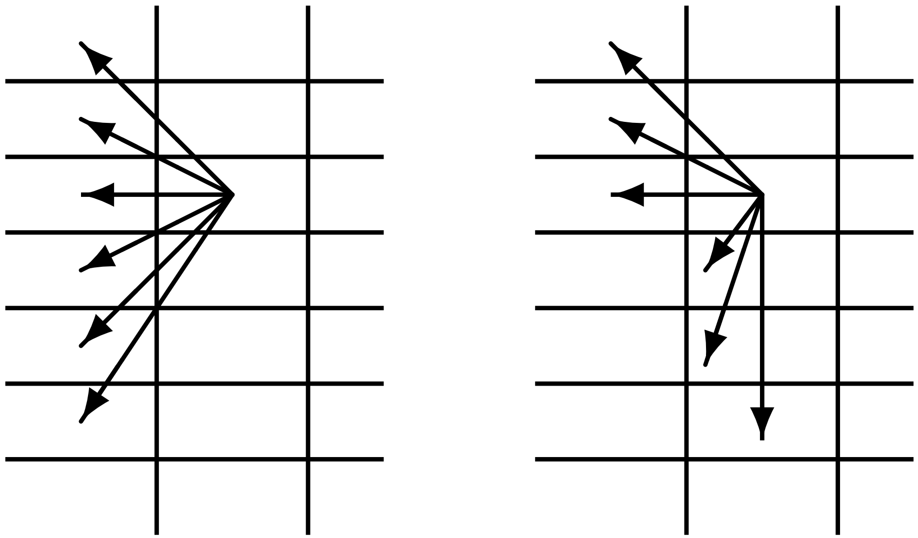
where:

$$Y_i = \text{uses}(\texttt{S}_i) \cup \Big(\bigcup_{\texttt{S}_j \in \text{succ}(\texttt{S}_i)} X_j - \text{defs}(\texttt{S}_i) \Big)$$

	$f^3(\perp)$	$f^4(\perp)$	$f^5(\perp)$
S1	{}	{}	{}
S2	{}	{R1}	{R1}
S3	{R1}	{R1}	{R1}
S4	{R1}	{R1}	{R1,R2,R3}
S5	{R1}	{R1,R2,R3}	{R1,R2,R3,R4}
S6	{R1,R2,R3,R5}	{R1,R2,R3,R4,R5}	{R1,R2,R3,R4,R5}
S7	{R1,R2,R3,R4}	{R1,R2,R3,R4}	{R1,R2,R3,R4}
S8	{R1,R2,R4}	{R1,R2,R4}	{R1,R2,R4}
S9	{R1,R3,R4}	{R1,R3,R4}	{R1,R3,R4}
S10	{R1,R4}	{R1,R4}	{R1,R4}
S11	{R1}	{R1}	{R1}
S12	{R1}	{R1}	{R1,R2,R3}
S13	{R1}	{R1,R2,R3}	{R1,R2,R3,R4}

	$f^6(\perp)$	$f^7(\perp)$	$f^8(\perp)$
S1	{}	{}	{}
S2	{R1}	{R1}	{R1}
S3	{R1,R2}	{R1,R2}	{R1,R2}
S4	{R1,R2,R3}	{R1,R2,R3}	{R1,R2,R3}
S5	{R1,R2,R3,R4}	{R1,R2,R3,R4}	{R1,R2,R3,R4}
S6	{R1,R2,R3,R4,R5}	{R1,R2,R3,R4,R5}	{R1,R2,R3,R4,R5}
S7	{R1,R2,R3,R4}	{R1,R2,R3,R4}	{R1,R2,R3,R4}
S8	{R1,R2,R4}	{R1,R2,R4}	{R1,R2,R3,R4}
S9	{R1,R3,R4}	{R1,R3,R4}	{R1,R2,R3,R4}
S10	{R1,R4}	{R1,R2,R3,R4}	{R1,R2,R3,R4}
S11	{R1,R2,R3}	{R1,R2,R3,R4}	{R1,R2,R3,R4}
S12	{R1,R2,R3,R4}	{R1,R2,R3,R4}	{R1,R2,R3,R4}
S13	{R1,R2,R3,R4}	{R1,R2,R3,R4}	{R1,R2,R3,R4}

A turbo fixed point technique:



The improved function is:

$$f_\Delta(X_1, X_2, \ldots, X_{13}) = (Y_1, Y_2, \ldots, Y_{13})$$

where:

$$Y_i = \text{uses}(S_i) \cup \left(\bigcup_{S_j \in \text{succ}(S_i)} Z_j - \text{defs}(S_i) \right)$$

$$Z_j = \begin{cases} Y_j & \text{if } j > i \\ X_j & \text{otherwise} \end{cases}$$

Improved fixed point computation:

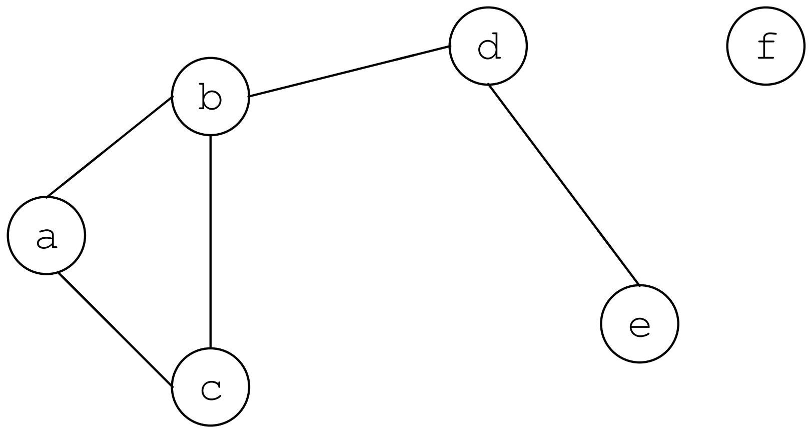
	$\perp$	$f_{\Delta}(\perp)$	$f^2_{\Delta}(\perp)$	$f^3_{\Delta}(\perp)$
S1	{}	{}	{}	{}
S2	{}	{}	{R1}	{R1}
S3	{}	{}	{R1,R2}	{R1,R2}
S4	{}	{}	{R1,R2,R3}	{R1,R2,R3}
S5	{}	{R1}	{R1,R2,R3,R4}	{R1,R2,R3,R4}
S6	{}	{R5}	{R1,R2,R3,R4,R5}	{R1,R2,R3,R4,R5}
S7	{}	{}	{R1,R2,R3,R4}	{R1,R2,R3,R4}
S8	{}	{R1,R2}	{R1,R2,R4}	{R1,R2,R3,R4}
S9	{}	{R1,R3}	{R1,R3,R4}	{R1,R2,R3,R4}
S10	{}	{R1,R4}	{R1,R4}	{R1,R2,R3,R4}
S11	{}	{R1}	{R1}	{R1,R2,R3,R4}
S12	{}	{R1}	{R1}	{R1,R2,R3,R4}
S13	{}	{}	{R1}	{R1,R2,R3,R4}

Number of iterations is down from 8 to 3.

Liveness analysis is used for register allocation in optimizing compilers.

In the basic block case, reduce spills to those variables that are only in registers *and* live.

In the intraprocedural case, construct a graph whose nodes are variables:



and where edges connect nodes that are live at the same time.

Register allocation is now reduced to finding a minimal graph coloring:

```
{ {a,d,f}, {b,e}, {c} }
```

and assigning a register to each color.

Liveness analysis is a *backwards* analysis, since we unravel from the future towards the past.

An example of a *forwards* analysis is constant propagation:

```
S1:  mov 3,R1         {(R0,?),(R1,?),(R2,?),(R3,?)}
      |
      v
S2:  mov 4,R2         {(R0,?),(R1,3),(R2,?),(R3,?)}
      |
      v
S3:  add R1,R2,R3     {(R0,?),(R1,3),(R2,4),(R3,?)}
      |
      v
S4:  mov R3,R0        {(R0,?),(R1,3),(R2,4),(R3,7)}
      |
      v
S5:  return           {(R0,7),(R1,3),(R2,4),(R3,7)}
```

A basic static analysis of JOOS and other object-oriented languages is *type inference*.

Given an expression, what are the possible classes of the objects to which it may evaluate?

The exact answer is undecidable, so we must conservatively approximate:

- we will accept a set that is too large;
- we want it as small as possible; and
- a trivial answer includes all classes.

This analysis is interprocedural and requires access to the whole program.

Possible uses of type inference:

- inline methods when there is only one possible receiver;
- eliminate run-time checks that can be decided statically;
- remove code that is never executed; and
- approximate the control flow graph to enable other static analyses.

In each case, smaller inferred sets will give better results.

The constraint technique:

- assign a variable $[\![\mathtt{E}]\!]$ to each occurrence of an expression $\mathtt{E}$;
- assign a variable $[\![\mathtt{m}]\!]$ to each occurrence of a method $\mathtt{m}$;
- the variables range over the set of all classes $C = \{\mathtt{C}_1, \mathtt{C}_2, \ldots, \mathtt{C}_n\}$;
- each parse tree node generates a local constraint on the variables; and
- the global minimal solution of these constraints is finally computed.

Again, we must compute a minimal fixed point in a finite lattice.

Each constraint models the flow of objects:

- the assignment "`i = E`" yields: $[\![\mathtt{E}]\!] \subseteq [\![\mathtt{i}]\!]$;
- the creation "`new C()`" yields: $\{\mathtt{C}\} \subseteq [\![\mathtt{new\ C()}]\!]$;
- the cast "`C(E)`" yields:
  $\{\mathtt{C}\} \subseteq [\![\mathtt{C(E)}]\!]$;
- the constant "`this`" yields: $\{\mathtt{C}\} \subseteq [\![\mathtt{this}]\!]$,
  where `C` is the surrounding class; and
- the statement "`return E`" yields: $[\![\mathtt{E}]\!] \subseteq [\![\mathtt{m}]\!]$,
  where `m` is the surrounding method.

The method invocation:

$\texttt{E.m(E}_{\boldsymbol{1}}\texttt{,E}_{\boldsymbol{2}}\texttt{,...,E}_{\boldsymbol{k}}\texttt{)}$

yields the *conditional* constraints:

$$\texttt{C}_{\boldsymbol{i}} \in [\![\texttt{E}]\!] \Rightarrow \begin{cases} [\![\texttt{E}_{\boldsymbol{1}}]\!] \subseteq [\![\texttt{x}_{\boldsymbol{1}}]\!] \\ [\![\texttt{E}_{\boldsymbol{2}}]\!] \subseteq [\![\texttt{x}_{\boldsymbol{2}}]\!] \\ \vdots \\ [\![\texttt{E}_{\boldsymbol{k}}]\!] \subseteq [\![\texttt{x}_{\boldsymbol{k}}]\!] \\ [\![\texttt{m}]\!] \subseteq [\![\texttt{E.m(E}_{\boldsymbol{1}}\texttt{,E}_{\boldsymbol{2}}\texttt{,...,E}_{\boldsymbol{k}}\texttt{)}]\!] \end{cases}$$

whenever the class $\texttt{C}_{\boldsymbol{i}}$ implements a method named $\texttt{m}$ which accepts $\boldsymbol{k}$ arguments named $\texttt{x}_{\boldsymbol{1}}$, $\texttt{x}_{\boldsymbol{2}}$, ..., $\texttt{x}_{\boldsymbol{k}}$.

Since the constraint:

$$v \subseteq w$$

holds if and only if the equality:

$$w = v \cup w$$

does, we can rewrite a set of constraints into a function:

$$f : \mathcal{P}(C)^k \rightarrow \mathcal{P}(C)^k$$

such that fixed-points of f correspond to solutions to the constraints.

For the example constraints:

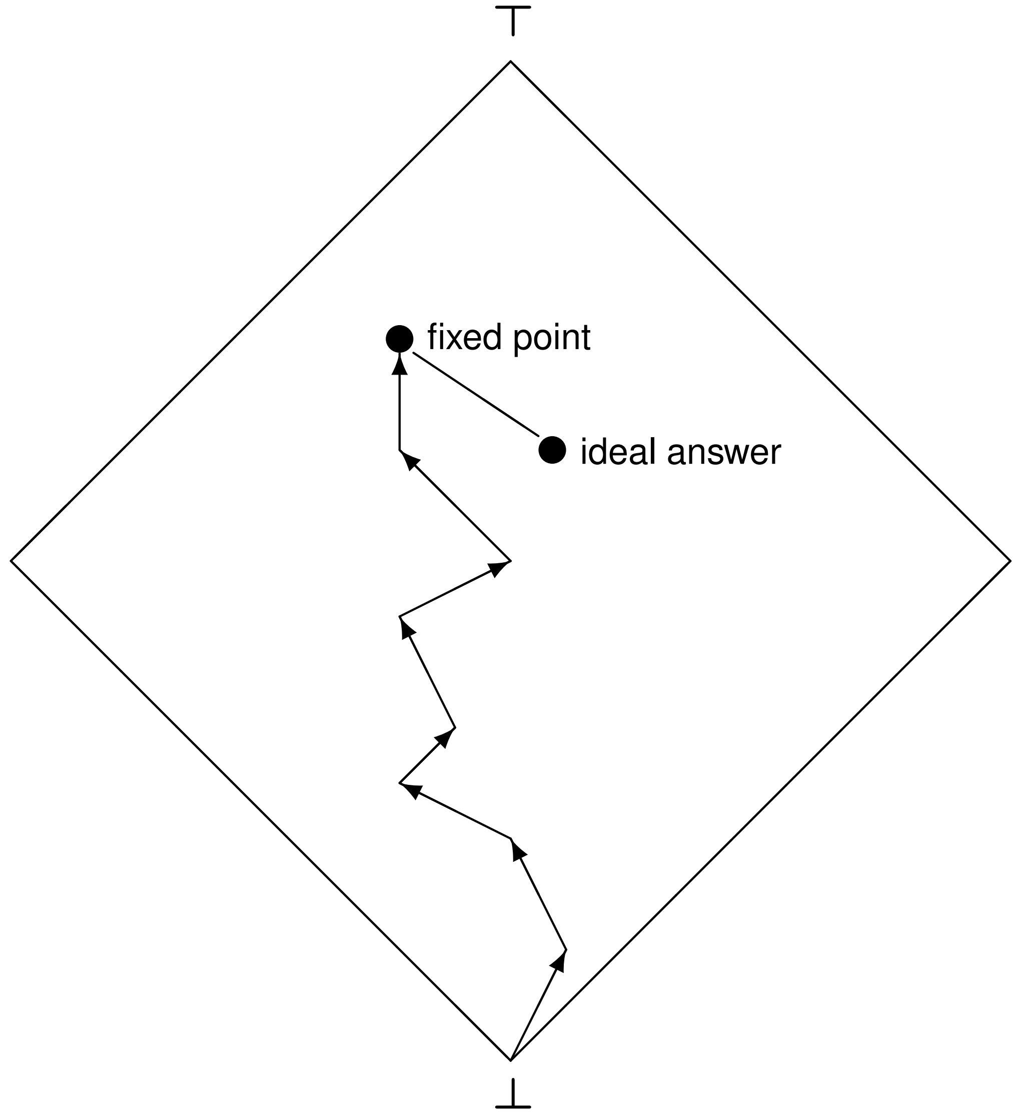
$$\boldsymbol{v_1} \subseteq \boldsymbol{v_2}$$
$$\mathsf{C}_\mathbf{3} \in \boldsymbol{v_2} \Rightarrow \boldsymbol{v_3} \subseteq \boldsymbol{v_1}$$
$$\{\mathsf{C}_\mathbf{7}\} \subseteq \boldsymbol{v_3}$$

we get the function:

$$f(X_1, X_2, X_3) = \begin{cases} (X_1 \cup X_3, X_1 \cup X_2, \{\mathsf{C}_7\} \cup X_3) & \text{if } \mathsf{C}_3 \in X_2 \\ (X_1, X_1 \cup X_2, \{\mathsf{C}_7\} \cup X_3) & \text{otherwise} \end{cases}$$

Solving the constraints:

- $\mathcal{P}(C)^k$ is a finite lattice;
- each function f is monotonic; and
- the least fixed point of f is the unique smallest solution of the constraints.



A tiny JOOS sketch:

```
public class A {
  public A() { super(); }
  public A id(A x) { return x; }
}

public class B extends A {
  public B() { super(); }
  public B me() { return (B)(new A()).id(this); }
}
```

The generated constraints are:

$[\![\texttt{x}]\!]_{\texttt{A}} \subseteq [\![\texttt{id}]\!]_{\texttt{A}}$
$[\![\texttt{x}]\!]_{\texttt{B}} \subseteq [\![\texttt{id}]\!]_{\texttt{B}}$
$[\![\texttt{(B)(new A()).id(this)}]\!] \subseteq [\![\texttt{me}]\!]$
$\{\texttt{B}\} \subseteq [\![\texttt{(B)(new A()).id(this)}]\!]$
$\{\texttt{B}\} \subseteq [\![\texttt{this}]\!]$
$\{\texttt{A}\} \subseteq [\![\texttt{new A()}]\!]$
$\texttt{A} \in [\![\texttt{new A()}]\!] \Rightarrow [\![\texttt{this}]\!] \subseteq [\![\texttt{x}]\!]_{\texttt{A}}$
$\texttt{A} \in [\![\texttt{new A()}]\!] \Rightarrow [\![\texttt{id}]\!]_{\texttt{A}} \subseteq [\![\texttt{(new A()).id(this)}]\!]$
$\texttt{B} \in [\![\texttt{new A()}]\!] \Rightarrow [\![\texttt{this}]\!] \subseteq [\![\texttt{x}]\!]_{\texttt{B}}$
$\texttt{B} \in [\![\texttt{new A()}]\!] \Rightarrow [\![\texttt{id}]\!]_{\texttt{B}} \subseteq [\![\texttt{(new A()).id(this)}]\!]$

The minimal solution is:

$[\![\texttt{new A()}]\!] = \{\texttt{A}\}$
$[\![\texttt{x}]\!]_{\texttt{A}} = [\![\texttt{id}]\!]_{\texttt{A}} = [\![\texttt{this}]\!] = [\![\texttt{(new A()).id(this)}]\!] = \{\texttt{B}\}$
$[\![\texttt{(B)(new A()).id(this)}]\!] = \{\texttt{B}\}$
$[\![\texttt{x}]\!]_{\texttt{B}} = [\![\texttt{id}]\!]_{\texttt{B}} = \{\}$

The generated code for the `me` method is:

```
.method public me()LB;
  .limit locals 1
  .limit stack 2
  new A
  dup
  invokenonvirtual A/<init>()V
  aload_0
  invokevirtual A/id(LA;)LA;
  checkcast B
  areturn
.end method
```

The information $[\![\texttt{new A().id(this)}]\!] = \{\texttt{B}\}$ eliminates the `checkcast` instruction.

That $[\![\texttt{new A()}]\!] = \{\texttt{A}\}$ is a singleton further allows inlining of the `id` method:

```
.method public me()LB;
  .limit locals 1
  .limit stack 1
  aload_0
  areturn
.end method
```

With type inference, many little methods become almost free.

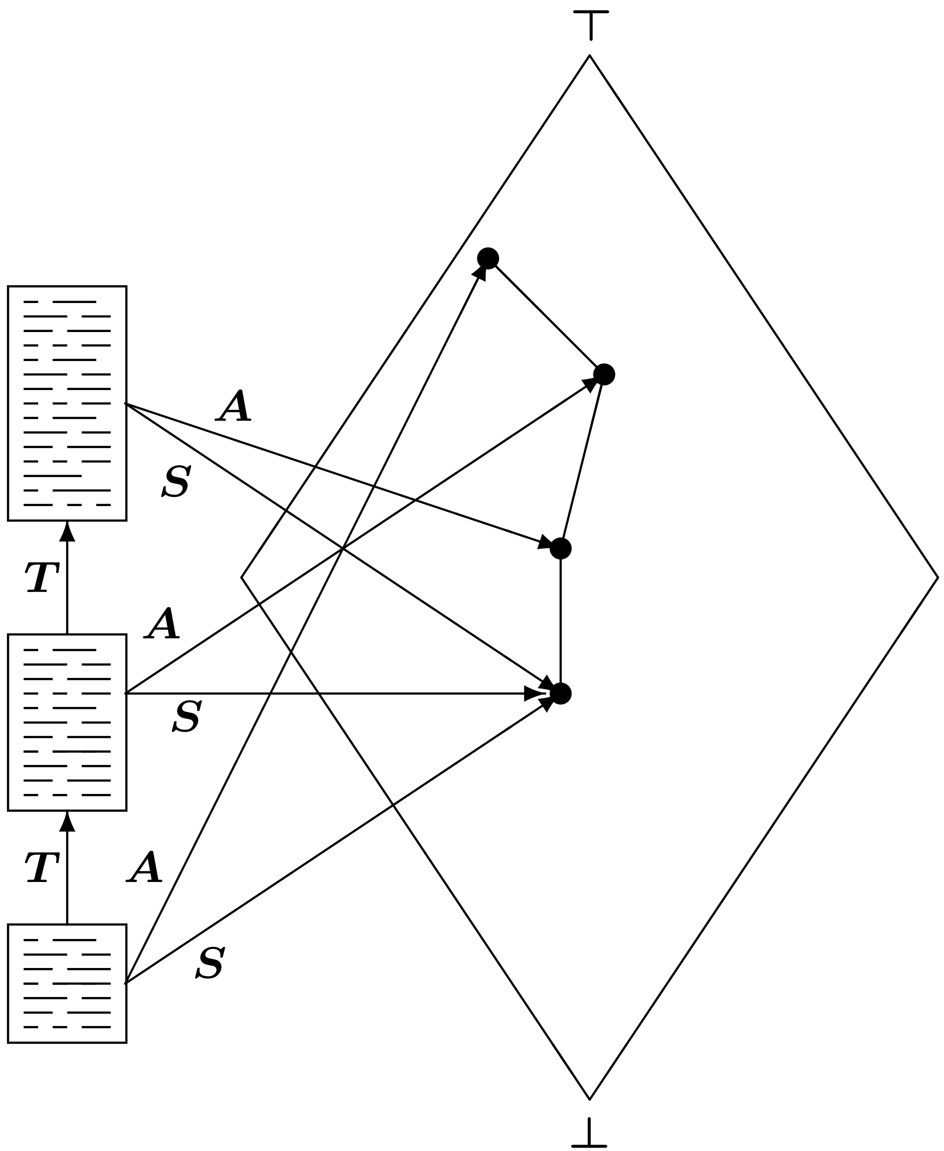
Improving analyses by transformations:

- let P be our set of programs;
- let $S : P \to D$ be an ideal static analysis (uncomputable); and
- let $T : P \to P$ be a program transformation that preserves the semantics.

Since S gives the ideal information, clearly $S(T(p)) = S(p)$ for all $p \in P$.

However, if $A : P \to D$ is a conservative approximation to S, then $A(T(p))$ may be different from $A(p)$, perhaps even better.

Transformations boost analyses:



The transformation T:

- may unfold the program to make it more explicit; or
- may itself be an optimization.