Type Checking

COMP 520: Compiler Design (4 credits)

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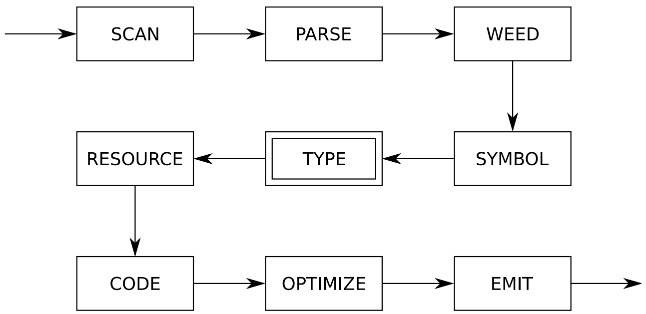


The *type checker* has severals tasks:

- determine the types of all expressions;
- check that values and variables are used correctly; and
- resolve certain ambiguities by transforming the program.

Some languages have no type checker.

A *type* describes possible values.

The JOOS types are:

- `void`: the empty type;
- `int`: the integers;
- `char`: the characters;
- `boolean`: `true` and `false`; and
- `C`: objects of class `C` or any subclass.

Plus an artificial type:

- `polynull`

which is the type of the polymorphic `null` constant.

A *type annotation*:

```
int x;
Cons y;
```

specifies an *invariant* about the **run-time** behavior:

- `x` will always contain an integer value; and
- `y` will always contain `null` or an object of type `Cons` or any subclass.

Usual type annotations are not very expressive as invariants.

You can have types without annotations, through type inference (e.g. in ML).

Types can be arbitrarily complex in theory.

A program is *type correct* if the type annotations are valid invariants.

Static type correctness is undecidable:

```
int x;
int j;

x = 0;
scanf("%i",&j);
TM(j);
x = true; // does this invalid type assignment happen?
```

where `TM(j)` simulates the `j`'th Turing machine on empty input.

The program is type correct if and only if `TM(j)` does not halt on empty input.

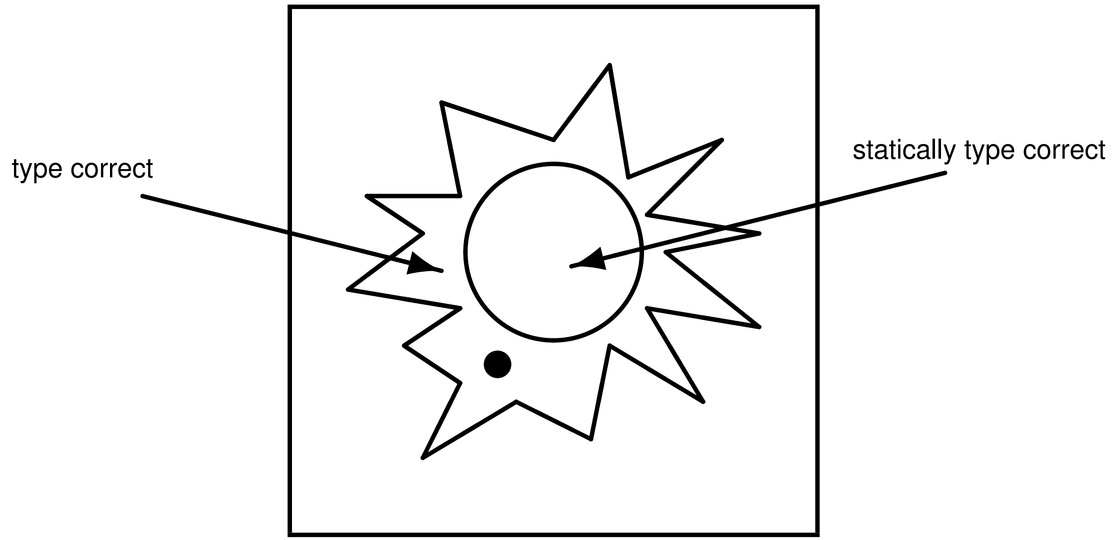
A program is *statically* type correct if it satisfies some type rules.

The type rules are chosen to be:

- simple to understand;
- efficient to decide; and
- conservative with respect to type correctness.

Type rules are rarely canonical.

Static type systems are necessarily flawed:



There is always *slack*, i.e. programs that are unfairly rejected by the type checker. Some are even quite useful.

```
int x;


x = 87;
if (false) x = true;
```

Type rules may be specified:

- in ordinary prose:

  The argument to the sqrt function must be of type int; the result is of type real.

- as constraints on type variables:

$$\texttt{sqrt(x)}: \quad [\![\texttt{sqrt(x)}]\!]=\texttt{real} \ \wedge \ [\![\texttt{x}]\!]=\texttt{int}$$

- as logical rules:

$$\frac{\mathcal{S} \vdash \texttt{x} : \texttt{int}}{\mathcal{S} \vdash \texttt{sqrt(x)} : \texttt{real}}$$

There are always three kinds:

1. declarations: introduction of variables;
2. propagations: expression type determines enclosing expression type; and
3. restrictions: expression type constrained by usage context

The judgement for statements:

$$\boldsymbol{L, C, M, V \vdash S}$$

means that $\boldsymbol{S}$ is statically type correct with:

- class library $\boldsymbol{L}$;
- current class $\boldsymbol{C}$;
- current method $\boldsymbol{M}$; and
- variables $\boldsymbol{V}$.

The judgement for expressions:

$$\boldsymbol{L, C, M, V \vdash E : \tau}$$

means that $\boldsymbol{E}$ is statically type correct and has type $\boldsymbol{\tau}$.

The tuple $\boldsymbol{L, C, M, V}$ is an abstraction of the symbol table.

From an implementation point of view ....

- A recursive traversal through the AST;
- Assuming we have a symbol table giving declared types;
- First type-checking the components; and
- then checking structure.

```
void typeImplementationCLASSFILE(CLASSFILE *c)
{ if (c!=NULL) {
    typeImplementationCLASSFILE(c->next);
    typeImplementationCLASS(c->class);
  }
}

void typeImplementationCLASS(CLASS *c)
{ typeImplementationCONSTRUCTOR(c->constructors,c);
  uniqueCONSTRUCTOR(c->constructors);
  typeImplementationMETHOD(c->methods,c);
}
```

The key parts are type checking statements and expressions

```
void typeImplementationSTATEMENT(STATEMENT *s, CLASS *this, TYPE *returntype)
{ if (s!=NULL) {
    switch (s->kind) {
      case skipK:
          break;
      case localK:
          break;
      case expK:
          typeImplementationEXP(s->val.expS,this);
          break;
      case returnK:
          ...
      case sequenceK:
          typeImplementationSTATEMENT(s->val.sequenceS.first,this,returntype);
          typeImplementationSTATEMENT(s->val.sequenceS.second,this,returntype);
          break;
      ...
}
```

Type checking expressions also stores the resulting type in the epxression node

```
void typeImplementationEXP(EXP *e, CLASS *this)
{ SYMBOL *s;
  TYPE *t;
  switch (e->kind) {
    case idK:
        e->type = typeVar(e->val.idE.idsym);
        break;
    case assignK:
        e->type = typeVar(e->val.assignE.leftsym);
        typeImplementationEXP(e->val.assignE.right,this);
        if (!assignTYPE(e->type,e->val.assignE.right->type)) {
          reportError("illegal assignment",e->lineno);
        }
        break;
    case orK:
        typeImplementationEXP(e->val.orE.left,this);
        typeImplementationEXP(e->val.orE.right,this);
        checkBOOL(e->val.orE.left->type,e->lineno);
        checkBOOL(e->val.orE.right->type,e->lineno);
        e->type = boolTYPE;
        break;
....
}
```

Type rules for statement sequence:

$$\frac{L, C, M, V \vdash S_1 \qquad L, C, M, V \vdash S_2}{L, C, M, V \vdash S_1\ S_2}$$

$$\frac{L, C, M, V[\mathrm{x} \mapsto \tau] \vdash S}{L, C, M, V \vdash \tau\ \ \mathrm{x}; S}$$

$V[\mathrm{x} \mapsto \tau]$ just says x maps to τ within V.

Corresponding JOOS source:

```
case sequenceK:
    typeImplementationSTATEMENT(s->val.sequenceS.first, class,returntype);
    typeImplementationSTATEMENT(s->val.sequenceS.second, class,returntype);
    break;
...

case localK:
    break;
```

Type rules for return statements:

$$\frac{\textit{return\_type}(L, C, M) = \texttt{void}}{L, C, M, V \vdash \texttt{return}}$$

$$\frac{L{,}C{,}M{,}V \vdash E : \tau \quad \textit{return\_type}(L{,}C{,}M) = \sigma \quad \sigma := \tau}{L, C, M, V \vdash \texttt{return}\ E}$$

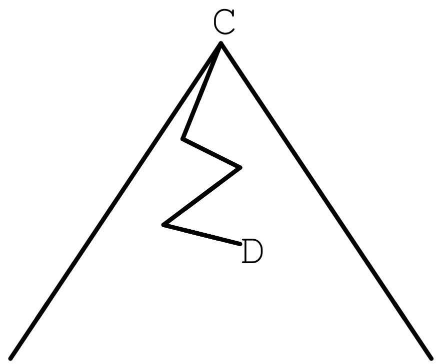
$\sigma := \tau$ just says something of type σ can be assigned something of type τ.

Corresponding JOOS source:

```
case returnK:
  if (s->val.returnS!=NULL) {
    typeImplementationEXP(s->val.returnS,class);
  }
  if (returntype->kind==voidK && s->val.returnS!=NULL) {
    reportError("return value not allowed",s->lineno);
  }
  if (returntype->kind!=voidK && s->val.returnS==NULL) {
    reportError("return value expected",s->lineno);
  }
  if (returntype->kind!=voidK && s->val.returnS!=NULL) {
    if (!assignTYPE(returntype,s->val.returnS->type)) {
      reportError("illegal type of expression", s->lineno);
    }
  }
  break;
```

Assignment compatibility:

- `int:=int`;
- `int:=char`;
- `char:=char`;
- `boolean:=boolean`;
- `C:=polynull`; and
- `C:=D`, if $D \leq C$.



Corresponding JOOS source:

```
int assignTYPE(TYPE *lhs, TYPE *rhs)
{ if (lhs->kind==refK && rhs->kind==polynullK) return 1;
  if (lhs->kind==intK && rhs->kind==charK) return 1;
  if (lhs->kind!=rhs->kind) return 0;
  if (lhs->kind==refK) return subClass(rhs->class,lhs->class);
  return 1;
}
```

Type rule for expression statements:

$$\frac{L, C, M, V \vdash E : \tau}{L, C, M, V \vdash E}$$

Corresponding JOOS source:

```
case expK:
    typeImplementationEXP(s->val.expS,class);
    break;
```

Type rule for `if`-statement:

$$\frac{L, C, M, V \vdash E : \texttt{boolean} \quad L, C, M, V \vdash S}{L, C, M, V \vdash \texttt{if (}E\texttt{)}\ S}$$

Corresponding JOOS source:

```
case ifK:
  typeImplementationEXP(s->val.ifS.condition,class);
  checkBOOL(s->val.ifS.condition->type,s->lineno);
  typeImplementationSTATEMENT(s->val.ifS.body, class, returntype);
  break;
```

Type rule for variables:

$$\frac{V(\mathrm{x}) = \tau}{L, C, M, V \vdash \mathrm{x} : \tau}$$

Corresponding JOOS source:

```
case idK:
    e->type = typeVar(e->val.idE.idsym);
    break;
```

Type rule for assignment:

$$\frac{L,C,M,V \vdash \mathrm{x} : \tau \quad L,C,M,V \vdash E : \sigma \quad \tau := \sigma}{L,C,M,V \vdash \mathrm{x}{=}E : \tau}$$

Corresponding JOOS source:

```
case assignK:
  e->type = typeVar(e->val.assignE.leftsym);
  typeImplementationEXP(e->val.assignE.right,class);
  if (!assignTYPE(e->type,e->val.assignE.right->type)) {
    reportError("illegal assignment",e->lineno);
  }
  break;
```

Type rule for minus:

$$\frac{L,C,M,V \vdash E_1 : \texttt{int} \qquad L,C,M,V \vdash E_2 : \texttt{int}}{L,C,M,V \vdash E_1 - E_2 : \texttt{int}}$$

Corresponding JOOS source:

```
case minusK:
    typeImplementationEXP(e->val.minusE.left,class);
    typeImplementationEXP(e->val.minusE.right,class);
    checkINT(e->val.minusE.left->type,e->lineno);
    checkINT(e->val.minusE.right->type,e->lineno);
    e->type = intTYPE;
    break;
```

Implicit integer cast:

$$\frac{L,C,M,V \vdash E : \texttt{char}}{L,C,M,V \vdash E : \texttt{int}}$$

Corresponding JOOS source:

```
int checkINT(TYPE *t, int lineno)
{ if (t->kind!=intK && t->kind!=charK) {
    reportError("int type expected",lineno);
    return 0;
  }
  return 1;
}
```

Type rule for equality:

$$\frac{\begin{array}{c} L,C,M,V \vdash E_1 : \tau_1 \\ L,C,M,V \vdash E_2 : \tau_2 \\ \tau_1 := \tau_2 \ \vee \ \tau_2 := \tau_1 \end{array}}{L,C,M,V \vdash E_1 \texttt{==} E_2 : \texttt{boolean}}$$

Corresponding JOOS source:

```
case eqK:
    typeImplementationEXP(e->val.eqE.left,class);
    typeImplementationEXP(e->val.eqE.right,class);
    if (!assignTYPE(e->val.eqE.left->type, e->val.eqE.right->type) &&
       !assignTYPE(e->val.eqE.right->type, e->val.eqE.left->type)) {
      reportError("arguments for == have wrong types",
                  e->lineno);
    }
    e->type = boolTYPE;
    break;
```

Type rule for `this`:

$$L,C,M,V \vdash \texttt{this} : C$$

Corresponding JOOS source:

```
case thisK:
    if (class==NULL) {
      reportError("'this' not allowed here",e->lineno);
    }
    e->type = classTYPE(class);
    break;
```

Type rule for cast:

$$\frac{L,C,M,V \vdash E : \tau \quad \tau \leq \mathtt{C} \vee \mathtt{C} \leq \tau}{L,C,M,V \vdash (\mathtt{C})\, E : \mathtt{C}}$$

Corresponding JOOS source:

```
case castK:
    typeImplementationEXP(e->val.castE.right,class);
    e->type = makeTYPEextref(e->val.castE.left,
                             e->val.castE.class);
    if (e->val.castE.right->type->kind!=refK &&
        e->val.castE.right->type->kind!=polynullK) {
       reportError("class reference expected",e->lineno);
    } else {
       if (e->val.castE.right->type->kind==refK &&
          !subClass(e->val.castE.class,
                    e->val.castE.right->type->class) &&
          !subClass(e->val.castE.right->type->class,
                    e->val.castE.class)) {
         reportError("cast will always fail",e->lineno);
       }
    }
    break;
```

Type rule for `instanceof`**:**

$$\frac{L,C,M,V \vdash E : \tau \quad \tau \leq \texttt{C} \vee \texttt{C} \leq \tau}{L,C,M,V \vdash E\ \texttt{instanceof C} : \texttt{boolean}}$$

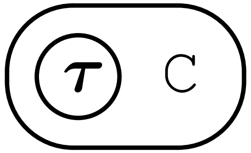
Corresponding JOOS source:

```
case instanceofK:
  typeImplementationEXP(e->val.instanceofE.left,class);
  if (e->val.instanceofE.left->type->kind!=refK) {
    reportError("class reference expected",e->lineno);
  }
  if (!subClass(e->val.instanceofE.left->type->class,
              e->val.instanceofE.class) &&
     !subClass(e->val.instanceofE.class,
              e->val.instanceofE.left->type->class)) {
    reportError("instanceof will always fail",e->lineno);
  }
  e->type = boolTYPE;
  break;
```

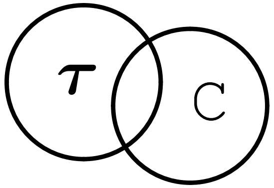
Why the predicate?:

$$\boldsymbol{\tau} \leq \texttt{C} \vee \texttt{C} \leq \boldsymbol{\tau}$$

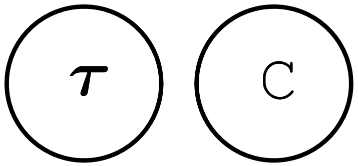
for "`(C)` $\boldsymbol{E}$" and "$\boldsymbol{E}$ `instanceof C`"?



succeeds $\boldsymbol{\tau} \leq \texttt{C}$



really useful $\texttt{C} \leq \boldsymbol{\tau}$



fails $\boldsymbol{\tau} \not\leq \texttt{C} \wedge \texttt{C} \not\leq \boldsymbol{\tau}$

Circle denotes type and all its subtypes. For instance, the following would fail to type check, as no subtype of `List` can ever be a subtype of the final (!) class `String`:

```
List l;
if(l instanceof String) ...
```

Type rule for method invocation:

$$\frac{\begin{array}{l} L, C, M, V \vdash E : \sigma \wedge \sigma \in L \\ \exists\, \rho\colon \sigma \leq \rho \wedge \mathtt{m} \in \textit{methods}(\rho) \\ \neg \textit{static}(\mathtt{m}) \\ L, C, M, V \vdash E_i : \sigma_i \\ \textit{argtype}(L, \rho, \mathtt{m}, i) := \sigma_i \\ \textit{return\_type}(L, \rho, \mathtt{m}) = \tau \end{array}}{L, C, M, V \vdash E\,\mathtt{.m(}E_1, \ldots, E_n\mathtt{)} : \tau}$$

Corresponding JOOS source:

```
case invokeK:
  t = typeImplementationRECEIVER(
        e->val.invokeE.receiver,class);
  typeImplementationARGUMENT(e->val.invokeE.args,class);
  if (t->kind!=refK) {
    reportError("receiver must be an object",e->lineno);
    e->type = polynullTYPE;
  } else {
    s = lookupHierarchy(e->val.invokeE.name,t->class);
    if (s==NULL || s->kind!=methodSym) {
      reportStrError("no such method called %s",
                     e->val.invokeE.name,e->lineno);
      e->type = polynullTYPE;
    } else {
      e->val.invokeE.method = s->val.methodS;
      if (s->val.methodS.modifier==modSTATIC) {
        reportStrError(
              "static method %s may not be invoked",
              e->val.invokeE.name,e->lineno);
      }
      typeImplementationFORMALARGUMENT(
         s->val.methodS->formals,
         e->val.invokeE.args,e->lineno);
      e->type = s->val.methodS->returntype;
    }
  }
  break;
```

Type rule for constructor invocation:

$$\frac{\begin{array}{l} L, C, M, V \vdash E_i : \sigma_i \\ \exists\, \vec{\tau}:\ \textit{constructor}(L, \mathtt{C}, \vec{\tau}) \wedge \\ \qquad \vec{\tau} := \vec{\sigma} \wedge \\ \qquad (\forall \vec{\gamma}:\ \ \textit{constructor}(L, \mathtt{C}, \vec{\gamma}) \wedge \vec{\gamma} := \vec{\sigma} \\ \qquad\qquad\quad \Downarrow \\ \qquad\qquad\quad \vec{\gamma} := \vec{\tau} \\ \qquad) \end{array}}{L, C, M, V \vdash \mathtt{new\ C(}E_1, \ldots, E_n\mathtt{)} : \mathtt{C}}$$

Corresponding JOOS source:

```
case newK:
    if (e->val.newE.class->modifier==modABSTRACT) {
       reportStrError("illegal abstract constructor %s",
                      e->val.newE.class->name,
                      e->lineno);
    }
    typeImplementationARGUMENT(e->val.newE.args,this);
    e->val.newE.constructor =
      selectCONSTRUCTOR(e->val.newE.class->constructors,
                        e->val.newE.args,
                        e->lineno);
    e->type = classTYPE(e->val.newE.class);
    break;
```

Simple example of an ambiguous constructor call

```
public class AmbConst
{ AmbConst(String s, Object o)
   { }

  AmbConst(Object o, String s)
   { }

  public static void main(String args[])
    { Object o = new AmbConst("abc","def");
    }
}
```

```
> javac AmbConst.java
AmbConst.java:9: error: reference to AmbConst is ambiguous
    { Object o  = new AmbConst("abc","def");
                  ^
  both constructor AmbConst(String,Object) in AmbConst and
       constructor AmbConst(Object,String) in AmbConst match
1 error
```

Different kinds of type rules are:

- *axioms*:

$$\boldsymbol{L, C, M, V} \vdash \texttt{this} : \boldsymbol{C}$$

- *predicates*:

$$\boldsymbol{\tau} \leq \texttt{C} \lor \texttt{C} \leq \boldsymbol{\tau}$$

- *inferences*:

$$\frac{\boldsymbol{L,C,M,V} \vdash \boldsymbol{E_1} : \texttt{int} \quad \boldsymbol{L,C,M,V} \vdash \boldsymbol{E_2} : \texttt{int}}{\boldsymbol{L,C,M,V} \vdash \boldsymbol{E_1}\text{-}\boldsymbol{E_2} : \texttt{int}}$$

A *type proof* is a tree in which:

- nodes are inferences; and
- leaves are axioms or true predicates.

A program is statically type correct *iff* it is the root of some type proof.

A type proof is just a trace of a successful run of the type checker.

An example type proof:

$$\dfrac{\dfrac{\dfrac{\boldsymbol{V}[\mathtt{x}\mapsto\mathtt{A}][\mathtt{y}\mapsto\mathtt{B}](\mathtt{y})=\mathtt{B}}{\boldsymbol{\mathcal{S}}\vdash\mathtt{y}:\mathtt{B}} \quad \dfrac{\dfrac{\boldsymbol{V}[\mathtt{x}\mapsto\mathtt{A}][\mathtt{y}\mapsto\mathtt{B}](\mathtt{x})=\mathtt{A}}{\boldsymbol{\mathcal{S}}\vdash\mathtt{x}:\mathtt{A}} \quad \mathtt{A}\leq\mathtt{B}\vee\mathtt{B}\leq\mathtt{A}}{\boldsymbol{\mathcal{S}}\vdash\mathtt{(B)x}:\mathtt{B}} \quad \mathtt{B}:=\mathtt{B}}{\dfrac{\boldsymbol{L},\boldsymbol{C},\boldsymbol{M},\boldsymbol{V}[\mathtt{x}\mapsto\mathtt{A}][\mathtt{y}\mapsto\mathtt{B}]\vdash\mathtt{y=(B)x}:\mathtt{B}}{\boldsymbol{L},\boldsymbol{C},\boldsymbol{M},\boldsymbol{V}[\mathtt{x}\mapsto\mathtt{A}][\mathtt{y}\mapsto\mathtt{B}]\vdash\mathtt{y=(B)x;}}}}{\dfrac{\boldsymbol{L},\boldsymbol{C},\boldsymbol{M},\boldsymbol{V}[\mathtt{x}\mapsto\mathtt{A}]\vdash\mathtt{B\ y;\ \ y=(B)x;}}{\boldsymbol{L},\boldsymbol{C},\boldsymbol{M},\boldsymbol{V}\vdash\mathtt{A\ x;\ \ B\ y;\ \ y=(B)x;}}}$$

where $\boldsymbol{\mathcal{S}} = \boldsymbol{L},\boldsymbol{C},\boldsymbol{M},\boldsymbol{V}[\mathtt{x}\mapsto\mathtt{A}][\mathtt{y}\mapsto\mathtt{B}]$ and we assume that $\mathtt{B}\leq\mathtt{A}$.

Type rules for plus:

$$\frac{L,C,M,V \vdash E_1 : \texttt{int} \quad L,C,M,V \vdash E_2 : \texttt{int}}{L,C,M,V \vdash E_1 + E_2 : \texttt{int}}$$

$$\frac{L,C,M,V \vdash E_1 : \texttt{String} \quad L,C,M,V \vdash E_2 : \tau}{L,C,M,V \vdash E_1 + E_2 : \texttt{String}}$$

$$\frac{L,C,M,V \vdash E_1 : \tau \quad L,C,M,V \vdash E_2 : \texttt{String}}{L,C,M,V \vdash E_1 + E_2 : \texttt{String}}$$

The operator $+$ is *overloaded*.

Corresponding JOOS source:

```
case plusK:
    typeImplementationEXP(e->val.plusE.left,class);
    typeImplementationEXP(e->val.plusE.right,class);
    e->type = typePlus(e->val.plusE.left,
                       e->val.plusE.right,e->lineno);
    break;
.
.
.

TYPE *typePlus(EXP *left, EXP *right, int lineno)
{ if (equalTYPE(left->type,intTYPE) &&
      equalTYPE(right->type,intTYPE)) {
    return intTYPE;
  }
  if (!equalTYPE(left->type,stringTYPE) &&
      !equalTYPE(right->type,stringTYPE)) {
    reportError("arguments for + have wrong types",
                lineno);
  }
  left->tostring = 1;
  right->tostring = 1;
  return stringTYPE;
}
```

A *coercion* is a conversion function that is inserted automatically by the compiler.

The code:

```
"abc" + 17 + x
```

is transformed into:

```
"abc" + (new Integer(17).toString()) + x.toString()
```

What effect would a rule like:

$$\frac{\boldsymbol{L,C,M,V \vdash E_1 : \tau} \quad \boldsymbol{L,C,M,V \vdash E_2 : \sigma}}{\boldsymbol{L,C,M,V \vdash E_1 {+} E_2 :} \texttt{ String}}$$

have on the type system if it were included?

The testing strategy for the type checker involves a further extension of the pretty printer, where the type of every expression is printed explicitly.

These types are then compared to a corresponding manual construction for a sufficient collection of programs.

Furthermore, every error message should be provoked by some test program.