

Points-to Analysis using BDDs

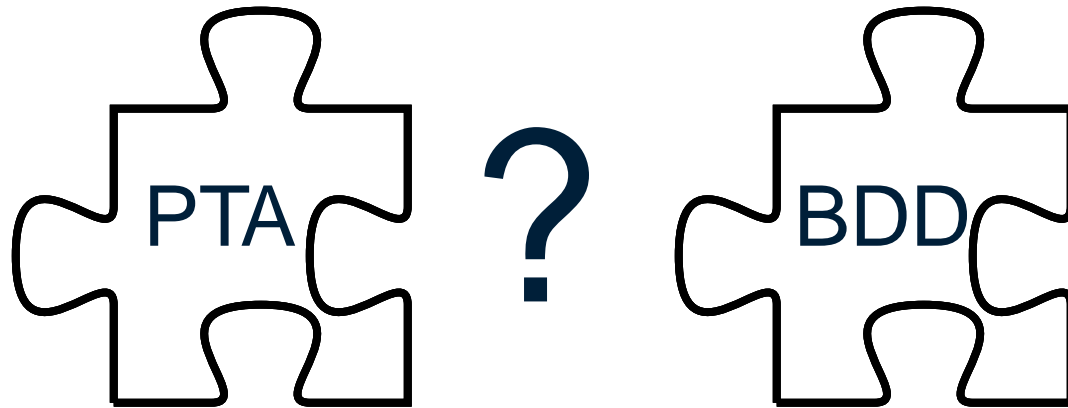
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Laurie Hendren, Navindra Umanee

Sable Research Group
McGill University
June 9th, 2003



Motivation

- Points-to analysis
 - requires representing many **large**, often **similar** sets
- Binary decision diagrams (BDDs)
 - provide compact representation of **large** sets with **similarities**



Background

■ Points-to analysis

- [Landi 92] [Andersen 94] [Emami 94] [Wilson 95]
[Steensgaard 96] [Shapiro 97] [Aiken 98] [Fähndrich 98]
[Ghiya 98] [Choi 99] [Das 00] [Hind 00] [Ruf 00]
[Sundaresan 00] [Tip 00] [Heintze 01] [Liang 01] [Rountev 01]
[Vivien 01] [Milanova 02] [Su 02] [Whaley 02] [Lhoták 03]
and more...

■ BDDs

- [Bryant 92] [Burch 94] and many, many more...

■ Program analysis using BDDs

- [Sias 00] [Manevich 02] [Ball 03]

Talk Outline

- **Introduction**
 - Points-to analysis
 - BDDs
- BDD-PTA algorithm
- Performance tuning
 - Bit ordering
 - Incrementalization
- Overall performance
- Conclusions and future work

Overview

- Designed a subset-based Java **points-to algorithm** using BDDs
- Implemented it using **BuDDy** BDD library
- Compared performance of **BDD-based solver** with **hand-tuned Spark solver** on identical input constraints
 - Spark solver is very efficient compared to other Java points-to solvers **[CC 03]**

BuDDy: provided by Jørn Lind-Nielsen at
<http://www.itu.dk/research/buddy>

Simple points-to analysis example

```
X: a = new O( ) ;
```

```
Y: b = new O( ) ;
```

```
Z: c = new O( ) ;
```

```
a = b ;
```

```
b = a ;
```

```
c = b ;
```

Points-to set:

```
{ }
```

Simple points-to analysis example

```
X: a = new O( ) ;
```

```
Y: b = new O( ) ;
```

```
Z: c = new O( ) ;
```

```
a = b ;
```

```
b = a ;
```

```
c = b ;
```

Points-to set:

```
{ (a,X) (b,Y) (c,Z) }
```

Simple points-to analysis example

```
X: a = new O( ) ;
```

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Y: b = new O( ) ;
```

```
Z: c = new O( ) ;
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```
a = b ;
```

```
b = a ;
```

```
c = b ;
```

Points-to set:

```
{ (a,X) (b,Y) (c,Z) (a,Y) }
```


Simple points-to analysis example

```
X: a = new O( ) ;
```

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```
Z: c = new O( ) ;
```

```
a = b ;
```

```
b = a ;
```

```
c = b ;
```

Points-to set:

```
{ (a,X) (b,Y) (c,Z) (a,Y) (b,X) }
```

Simple points-to analysis example

```
X: a = new O( ) ;
```

```
Y: b = new O( ) ;
```

```
Z: c = new O( ) ;
```

```
a = b ;
```

```
b = a ;
```

```
c = b ;
```

Points-to set:

```
{ (a,X) (b,Y) (c,Z) (a,Y) (b,X) (c,X) (c,Y) }
```

BDD representation

- A BDD is a compact representation of a set of bit strings
- We encode our analysis using bit strings:

$a \rightarrow 00$ $X \rightarrow 00$

$b \rightarrow 01$ $Y \rightarrow 01$

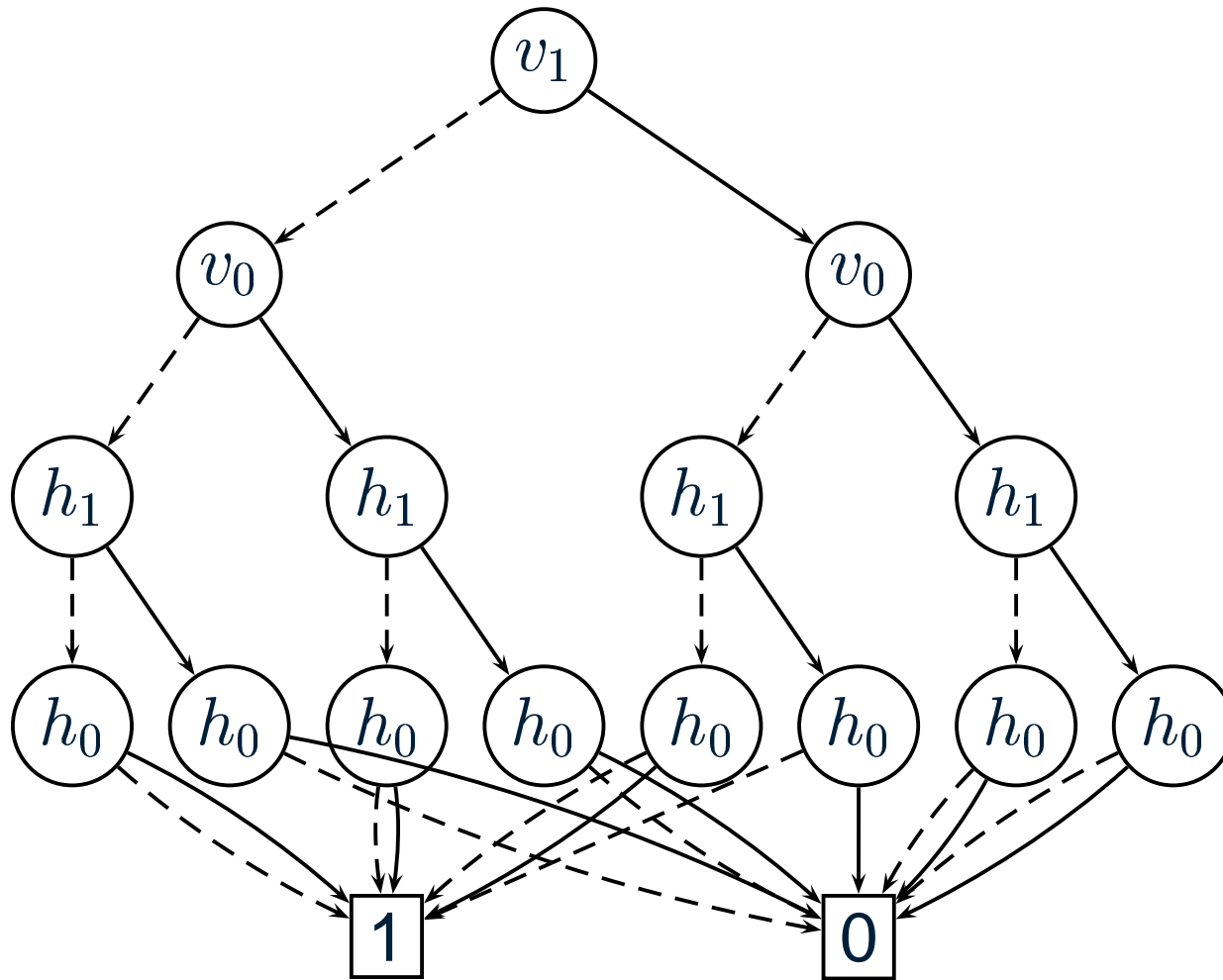
$c \rightarrow 10$ $Z \rightarrow 10$

Domains: V H

$v_1 v_0 h_1 h_0$

$(a, Y) \rightarrow 00 01$

BDD representation



a/X → 00

b/Y → 01

c/Z → 10

V H
 $v_1 v_0 h_1 h_0$

(a,X) 00 00

(a,Y) 00 01

(b,X) 01 00

(b,Y) 01 01

(c,X) 10 00

(c,Y) 10 01

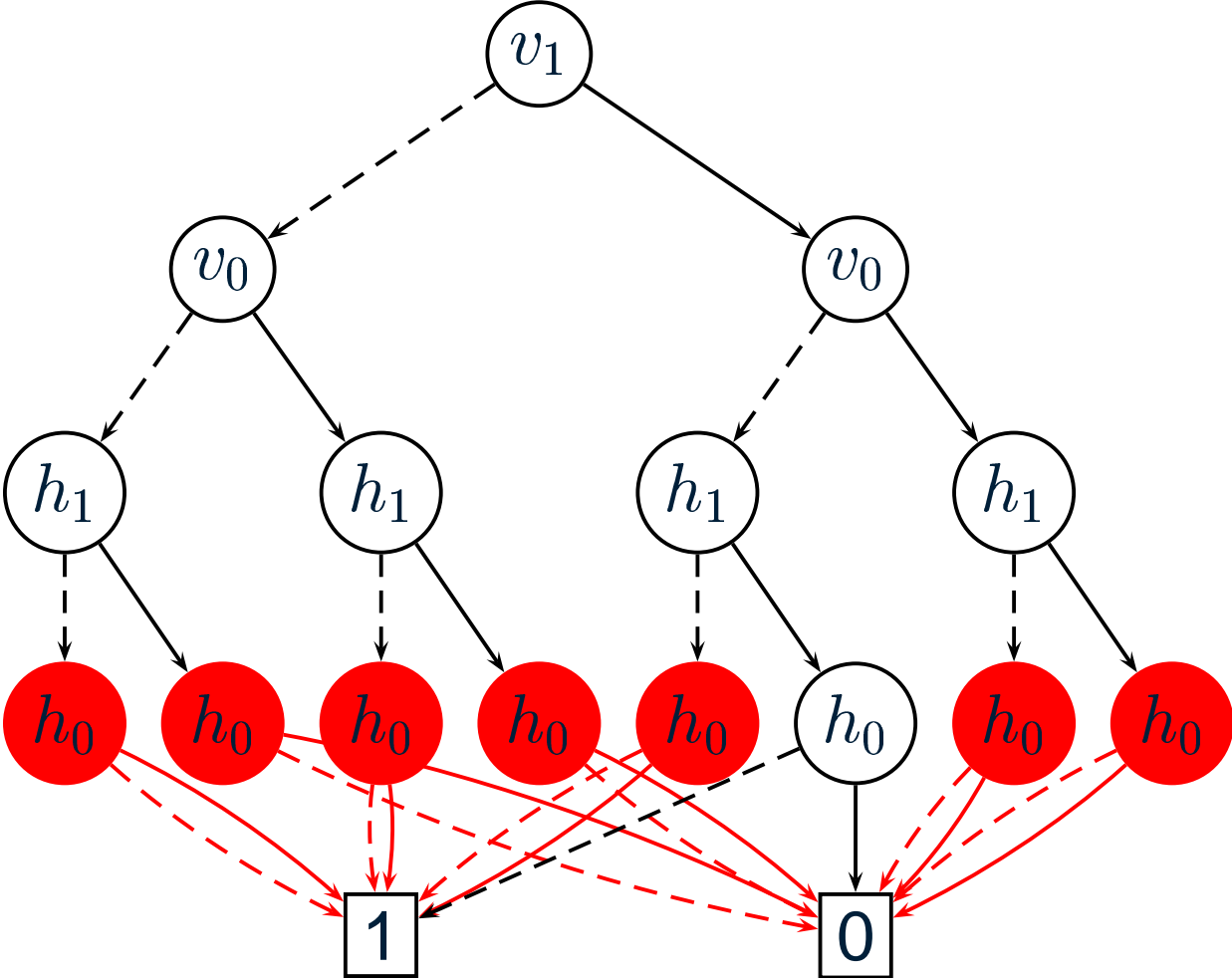
(c,Z) 10 10

BDD representation

a/X → 00

b/Y → 01

c/Z → 10



V H

$v_1 v_0 h_1 h_0$

(a,X) 00 00

(a,Y) 00 01

(b,X) 01 00

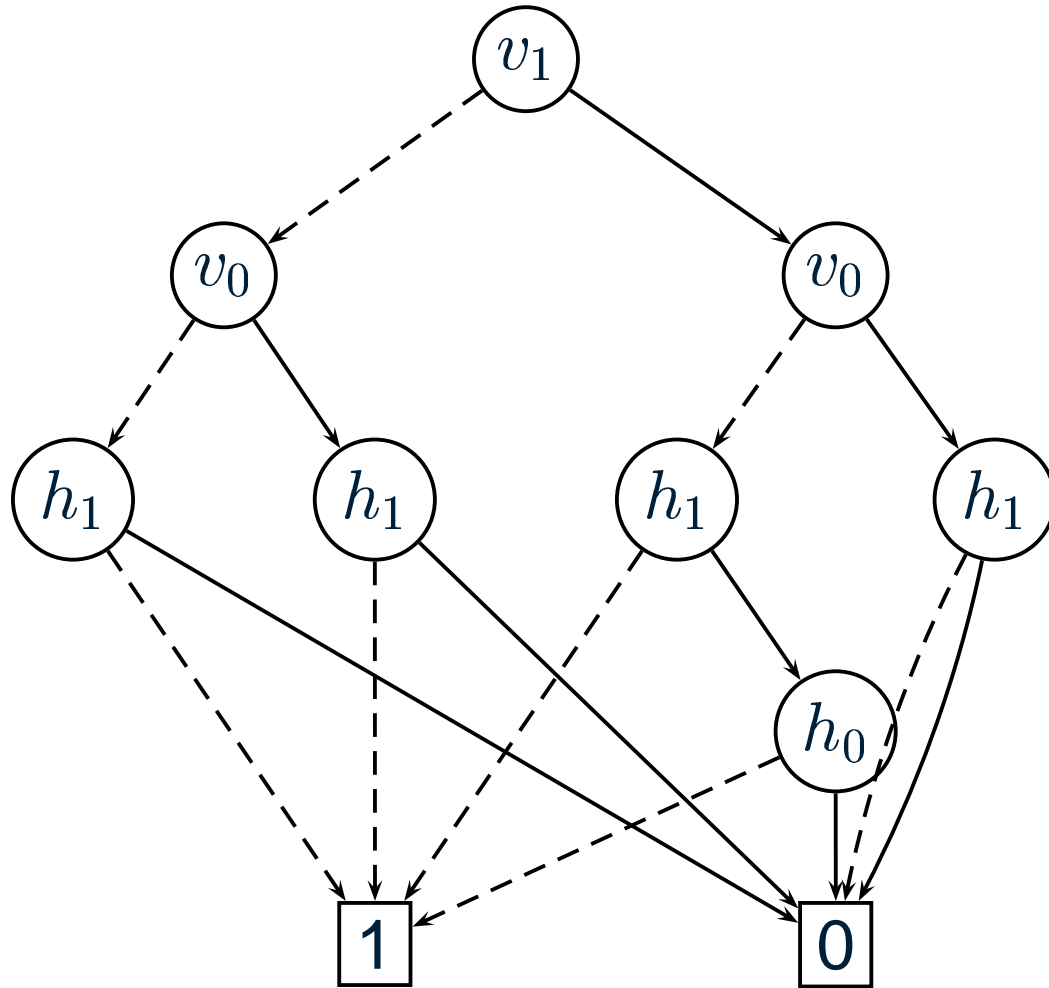
(b,Y) 01 01

(c,X) 10 00

(c,Y) 10 01

(c,Z) 10 10

BDD representation



$a/X \rightarrow 00$

$b/Y \rightarrow 01$

$c/Z \rightarrow 10$

V H

$v_1 v_0 h_1 h_0$

$(a, X) \ 00 \ 00$

$(a, Y) \ 00 \ 01$

$(b, X) \ 01 \ 00$

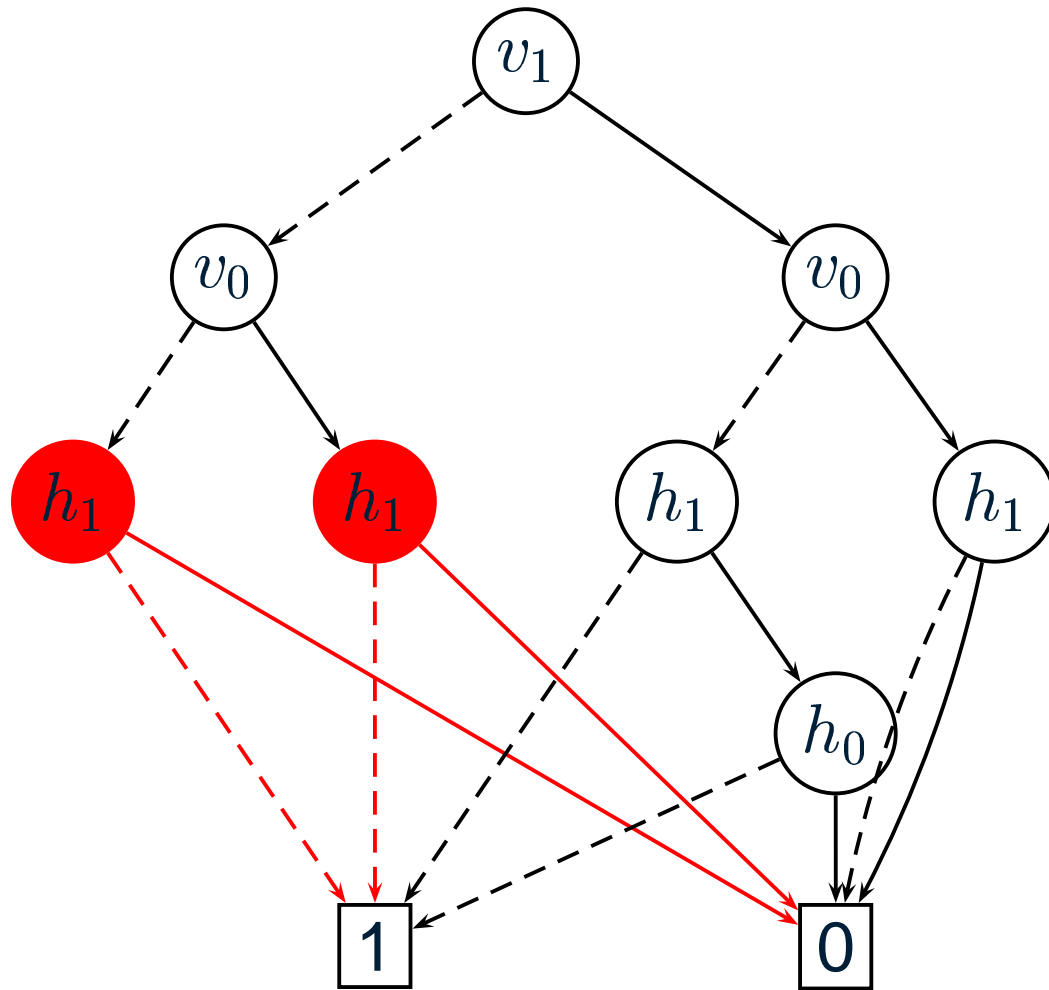
$(b, Y) \ 01 \ 01$

$(c, X) \ 10 \ 00$

$(c, Y) \ 10 \ 01$

$(c, Z) \ 10 \ 10$

BDD representation



a/X → 00

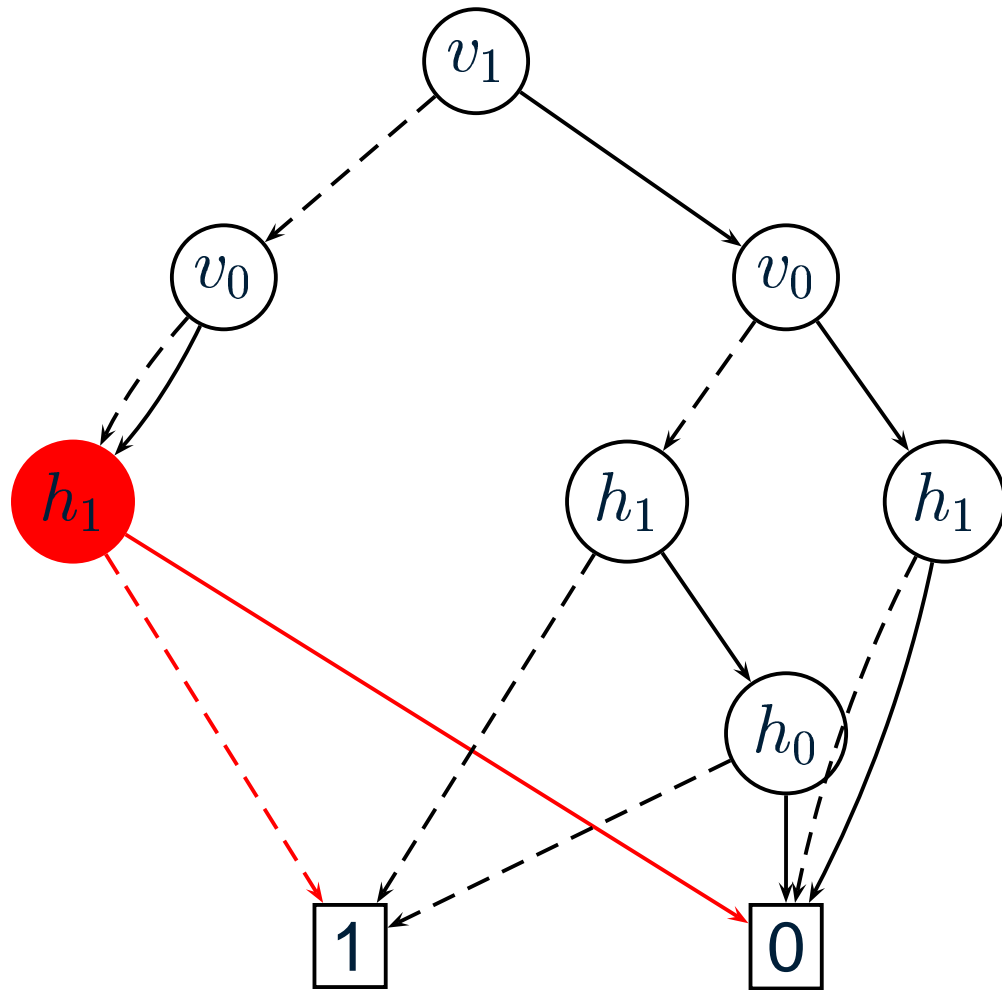
b/Y → 01

c/Z → 10

V H
 $v_1 v_0 h_1 h_0$

| | | |
|--------|----|----|
| (a, X) | 00 | 00 |
| (a, Y) | 00 | 01 |
| (b, X) | 01 | 00 |
| (b, Y) | 01 | 01 |
| (c, X) | 10 | 00 |
| (c, Y) | 10 | 01 |
| (c, Z) | 10 | 10 |

BDD representation



a/X \rightarrow 00

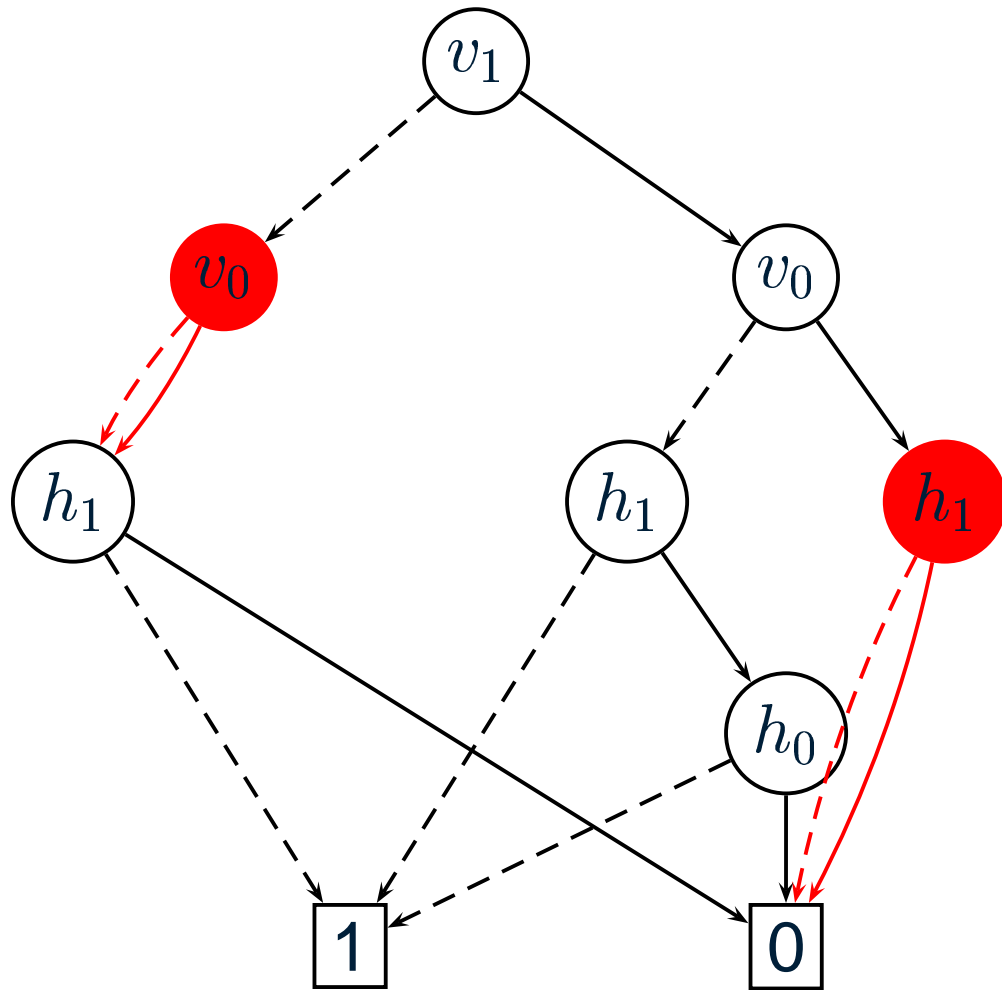
b/Y \rightarrow 01

c/Z \rightarrow 10

V H
 $v_1 v_0 h_1 h_0$

| | | |
|-------|----|----|
| (a,X) | 00 | 00 |
| (a,Y) | 00 | 01 |
| (b,X) | 01 | 00 |
| (b,Y) | 01 | 01 |
| (c,X) | 10 | 00 |
| (c,Y) | 10 | 01 |
| (c,Z) | 10 | 10 |

BDD representation



a/X → 00

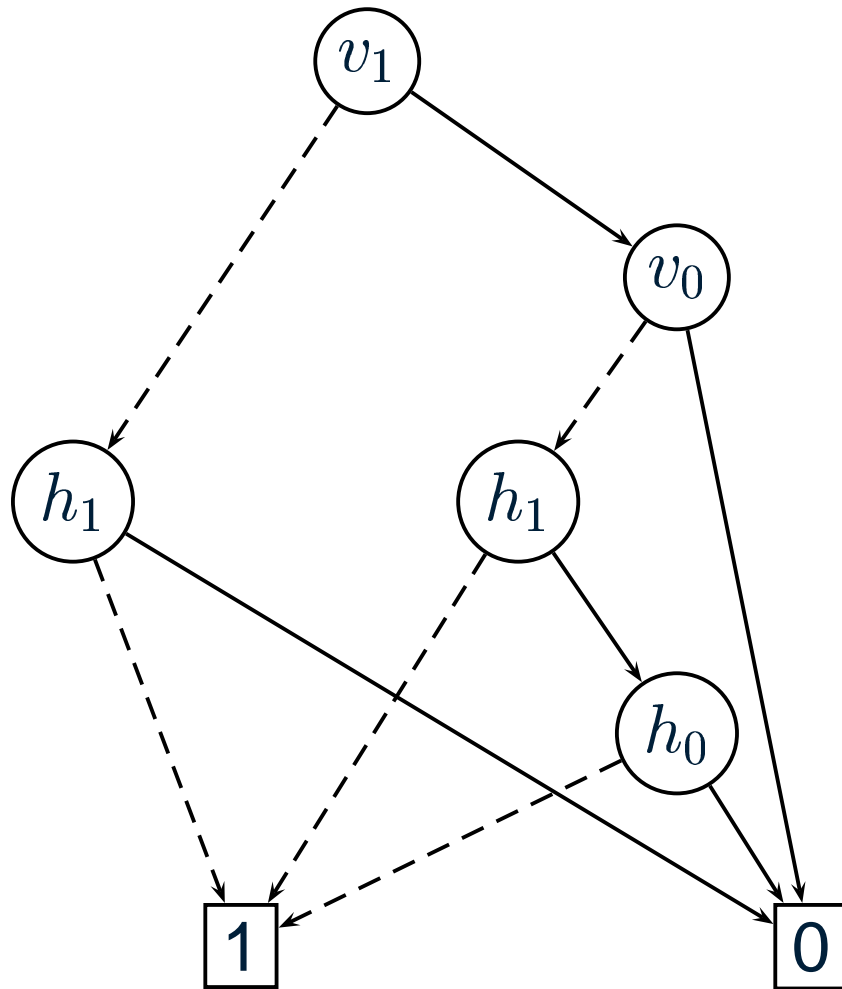
b/Y → 01

c/Z → 10

V H
 $v_1 v_0 h_1 h_0$

| | | |
|-------|----|----|
| (a,X) | 00 | 00 |
| (a,Y) | 00 | 01 |
| (b,X) | 01 | 00 |
| (b,Y) | 01 | 01 |
| (c,X) | 10 | 00 |
| (c,Y) | 10 | 01 |
| (c,Z) | 10 | 10 |

Reduced BDD representation



$a/X \rightarrow 00$

$b/Y \rightarrow 01$

$c/Z \rightarrow 10$

V H

$v_1 v_0 h_1 h_0$

$(a, X) \ 00 \ 00$

$(a, Y) \ 00 \ 01$

$(b, X) \ 01 \ 00$

$(b, Y) \ 01 \ 01$

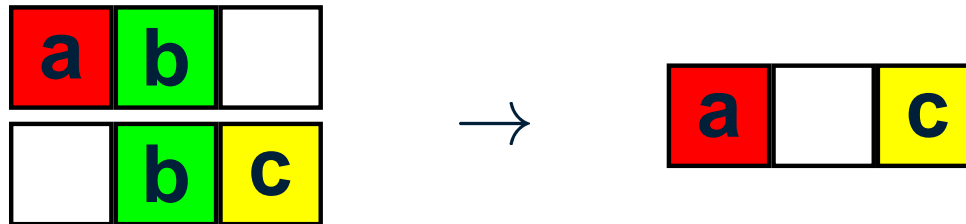
$(c, X) \ 10 \ 00$

$(c, Y) \ 10 \ 01$

$(c, Z) \ 10 \ 10$

BDD operations

- Set operations ($\cup, \cap, \setminus, \dots$)
- Relational product
($\{(a, c) \mid \exists b. (a, b) \in X \wedge (b, c) \in Y\}$)



- Replace –
changing bit order in a specific BDD



- Cost of operations proportional to number of nodes in BDD, not size of set represented

Propagating points-to sets

X: $a = \text{new } O();$ $a = b;$
 Y: $b = \text{new } O();$ $b = a;$
 Z: $c = \text{new } O();$ $c = b;$

(a,X) (b \rightarrow a)
 (b,Y) (a \rightarrow b)
 (c,Z) (b \rightarrow c)

| Domains | Points-to | Edges | New points-to |
|---------|-----------|-----------|---------------|
| V1 | a b c | b a b | |
| V2 | | a b c | |
| H1 | X Y Z | | |

Propagating points-to sets

```

X: a = new O( ) ;           a = b ;
Y: b = new O( ) ;           b = a ;
Z: c = new O( ) ;           c = b ;
  
```

```

      (a,X)   (b → a)
      (b,Y)   (a → b)
      (c,Z)   (b → c)
  
```

relprod

| Domains | Points-to | Edges | New points-to |
|---------|--------------|--------------|---------------|
| V1 | a b c | b a b | |
| V2 | | a b c | |
| H1 | X Y Z | | |

Propagating points-to sets

X: $a = \text{new } O();$ $a = b;$
 Y: $b = \text{new } O();$ $b = a;$
 Z: $c = \text{new } O();$ $c = b;$

(a,X) (b \rightarrow a)
 (b,Y) (a \rightarrow b)
 (c,Z) (b \rightarrow c)

relprod

| Domains | Points-to | Edges | New points-to |
|---------|-----------|-----------|---------------|
| V1 | a b c | b a b | |
| V2 | | a b c | b |
| H1 | X Y Z | | X |

Propagating points-to sets

```
X: a = new O( );      a = b;
Y: b = new O( );      b = a;
Z: c = new O( );      c = b;
```

```
(a,X)  (b → a)
(b,Y)  (a → b)
(c,Z)  (b → c)
```

relprod

| Domains | Points-to | Edges | New points-to |
|---------|-----------|-------|---------------|
| V1 | a b c | b a b | |
| V2 | | a b c | b |
| H1 | X Y Z | | X |

Propagating points-to sets

X: $a = \text{new } O();$ $a = b;$
 Y: $b = \text{new } O();$ $b = a;$
 Z: $c = \text{new } O();$ $c = b;$

(a,X) (b \rightarrow a)
 (b,Y) (a \rightarrow b)
 (c,Z) (b \rightarrow c)

relprod

| Domains | Points-to | Edges | New points-to |
|---------|--------------|---------------------|---------------|
| V1 | a b c | b a b | |
| V2 | | a b c | b |
| H1 | X Y Z | | X |

Propagating points-to sets

X: $a = \text{new } O();$ $a = b;$
 Y: $b = \text{new } O();$ $b = a;$
 Z: $c = \text{new } O();$ $c = b;$

(a,X) (b \rightarrow a)
 (b,Y) (a \rightarrow b)
 (c,Z) (b \rightarrow c)

relprod

| Domains | Points-to | Edges | New points-to |
|---------|--------------|---------------------|---------------------|
| V1 | a b c | b a b | |
| V2 | | a b c | b a c |
| H1 | X Y Z | | X Y Y |

Propagating points-to sets

X: $a = \text{new } O();$ $a = b;$
 Y: $b = \text{new } O();$ $b = a;$
 Z: $c = \text{new } O();$ $c = b;$

(a,X) (b → a)
 (b,Y) (a → b)
 (c,Z) (b → c)

| Domains | Points-to | Edges | New points-to |
|---------|-----------|-----------|---------------|
| V1 | a b c | b a b | |
| V2 | | a b c | b a c |
| H1 | X Y Z | | X Y Y |

Propagating points-to sets

```

X: a = new O();           a = b;
Y: b = new O();           b = a;
Z: c = new O();           c = b;
  
```

```

      (a,X)   (b → a)
      (b,Y)   (a → b)
      (c,Z)   (b → c)
  
```

replace

| Domains | Points-to | Edges | New points-to |
|---------|-----------|-------|---------------|
| V1 | a b c | b a b | |
| V2 | | a b c | b a c |
| H1 | X Y Z | | X Y Y |

Propagating points-to sets

```

X: a = new O();           a = b;
Y: b = new O();           b = a;
Z: c = new O();           c = b;
  
```

```

      (a,X)   (b → a)
      (b,Y)   (a → b)
      (c,Z)   (b → c)
  
```

replace

| Domains | Points-to | Edges | New points-to |
|---------|-----------|-------|---------------|
| V1 | a b c | b a b | b a c |
| V2 | | a b c | |
| H1 | X Y Z | | X Y Y |

Propagating points-to sets

X: $a = \text{new } O();$ $a = b;$
 Y: $b = \text{new } O();$ $b = a;$
 Z: $c = \text{new } O();$ $c = b;$

(a,X) (b \rightarrow a)
 (b,Y) (a \rightarrow b)
 (c,Z) (b \rightarrow c)

| Domains | Points-to | Edges | New points-to |
|---------|-----------|-------|---------------|
| V1 | a b c | b a b | b a c |
| V2 | | a b c | |
| H1 | X Y Z | | X Y Y |

Propagating points-to sets

X: $a = \text{new } O();$ $a = b;$
 Y: $b = \text{new } O();$ $b = a;$
 Z: $c = \text{new } O();$ $c = b;$

(a,X) (b \rightarrow a)
 (b,Y) (a \rightarrow b)
 (c,Z) (b \rightarrow c)

union

| Domains | Points-to | Edges | New points-to |
|---------|-----------|-------|---------------|
| V1 | a b c | b a b | b a c |
| V2 | | a b c | |
| H1 | X Y Z | | X Y Y |

Propagating points-to sets

X: $a = \text{new } O();$ $a = b;$
 Y: $b = \text{new } O();$ $b = a;$
 Z: $c = \text{new } O();$ $c = b;$

(a,X) (b \rightarrow a)
 (b,Y) (a \rightarrow b)
 (c,Z) (b \rightarrow c) union

| Domains | Points-to | Edges | New |
|---------|--------------------------------------|-----------|-----|
| V1 | a b c b a c | b a b | |
| V2 | | a b c | |
| H1 | X Y Z X Y Y | | |

Talk Outline

- Introduction
 - Points-to analysis
 - BDDs
- **BDD-PTA algorithm**
- Performance tuning
 - Bit ordering
 - Incrementalization
- Overall performance
- Conclusions and future work

BDDs used

- $edgeset \subseteq V1 \times V2$
simple assignments
($l_2 := l_1$)
- $stores \subseteq V1 \times (V2 \times FD)$
field stores
($l_2.f := l_1$)
- $loads \subseteq (V1 \times FD) \times V2$
field loads
($l_2 := l_1.f$)
- $pointsTo \subseteq V1 \times H1$
points-to relation for variables
(l points to o)
- $fieldPt \subseteq (H1 \times FD) \times H2$
points-to relation for object fields
($o_1.f$ points to o_2)
- 5 domains needed: $V1, V2, H1, H2, FD$

Overall algorithm

```
initialize
```

```
repeat
```

```
  repeat
```

```
    (1) process simple assignments
```

```
  until no change
```

```
    (2) process field stores
```

```
    (3) process field loads
```

```
until no change
```

Simple assignments ($l_2 := l_1$)

$$(1) \quad \frac{l_1 \rightarrow l_2 \quad o \in pt(l_1)}{o \in pt(l_2)}$$

```
newPt1: [V2xH1] =
  relprod( edgeSet: [V1xV2],
           pointsTo:[V1xH1],
           V1 );

newPt2: [V1xH1] =
  replace( newPt1: [V2xH1],
          V2ToV1 );

pointsTo:[V1xH1] =
  union( pointsTo:[V1xH1],
        newPt2: [V1xH1] );
```

Field stores ($q.f := l$)

$$(2) \quad \frac{o_2 \in pt(l) \quad l \rightarrow q.f \quad o_1 \in pt(q)}{o_2 \in pt(o_1.f)}$$

tmpRel1 : [(V2xFD)xH1] =

```
  relprod( stores: [V1x(V2xFD)],  
           pointsTo:[V1xH1],  
           V1 );
```

tmpRel2 : [(V1xFD)xH2] =

```
  replace( tmpRel1: [(V2xFD)xH1],  
           V2ToV1&H1ToH2 );
```

fieldPt : [(H1xFD)xH2] =

```
  relprod( tmpRel2: [(V1xFD)xH2],  
           pointsTo:[V1xH1],  
           V1 );
```

Field loads ($l := p.f$)

$$(3) \quad \frac{p.f \rightarrow l \quad o_1 \in pt(p) \quad o_2 \in pt(o_1.f)}{o_2 \in pt(l)}$$

```
tmpRel3:      [(H1xFD)xV2] =
  relprod( loads:      [(V1xFD)xV2],
           pointsTo:[V1xH1],
           V1      );

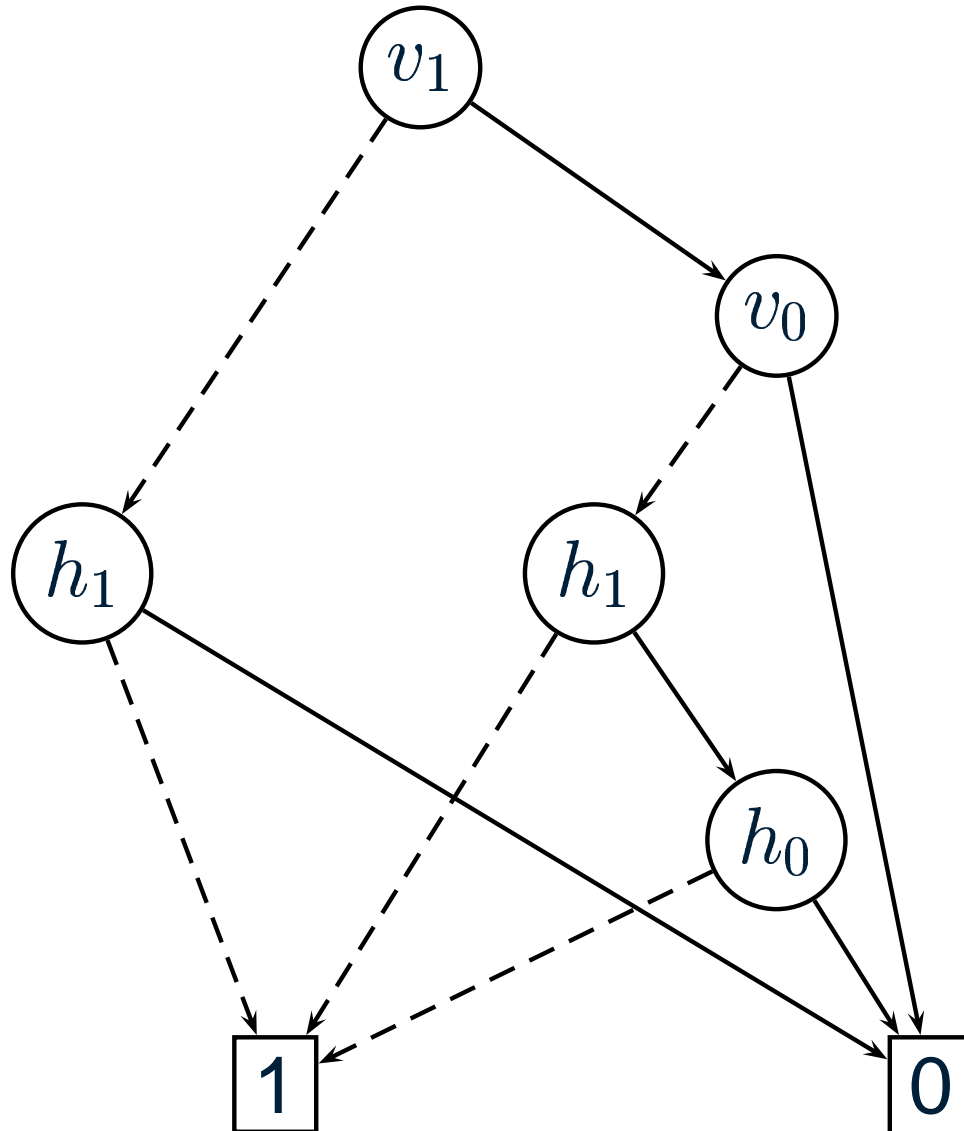
newPt4:      [V2xH2] =
  relprod( tmpRel3: [(H1xFD)xV2],
           fieldPt: [(H1xFD)xH2],
           H1xFD   );

newPt5:      [V1xH1] =
  replace( newPt4: [V2xH2],
           V2ToV1&H2ToH1 );
```

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- Introduction
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- Performance tuning
 - Bit ordering
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Bit ordering matters



a/X → 00

b/Y → 01

c/Z → 10

$v_1v_0h_1h_0$

(a,X) 0000

(a,Y) 0001

(b,X) 0100

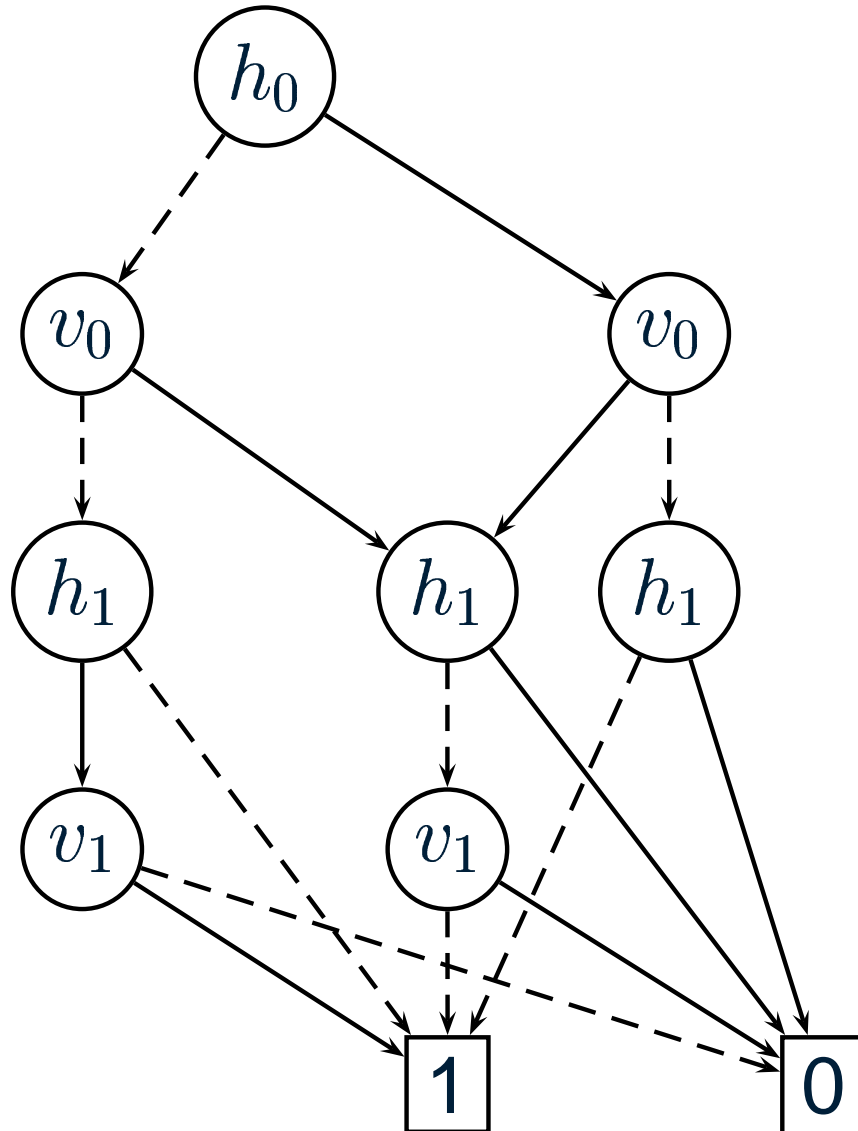
(b,Y) 0101

(c,X) 1000

(c,Y) 1001

(c,Z) 1010

Bit ordering matters



a/X → 00

b/Y → 10

c/Z → 01

$h_0 v_0 h_1 v_1$

(a,X) 0000

(a,Y) 1000

(b,X) 0100

(b,Y) 1100

(c,X) 0001

(c,Y) 1001

(c,Z) 0011

How to find a good ordering?

- BuDDy default is to interleave bits:

FD

| | | | |
|---|---|---|---|
| 3 | 2 | 1 | 0 |
|---|---|---|---|

V1

| | | | |
|---|---|---|---|
| 3 | 2 | 1 | 0 |
|---|---|---|---|

V2

| | | | |
|---|---|---|---|
| 3 | 2 | 1 | 0 |
|---|---|---|---|

H1

| | | | |
|---|---|---|---|
| 3 | 2 | 1 | 0 |
|---|---|---|---|

H2

| | | | |
|---|---|---|---|
| 3 | 2 | 1 | 0 |
|---|---|---|---|

| | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 3 | 3 | 3 | 3 | 3 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|

- Good heuristic for state machines in model checking
- Bad for points-to analysis: much too slow!

How to find a good ordering?

Where is most of the time spent?

$$(1) \quad \frac{l_1 \rightarrow l_2 \quad o \in pt(l_1)}{o \in pt(l_2)}$$

```
newPt1: [V2xH1] =  
  relprod( edgeSet: [V1xV2],  
           pointsTo:[V1xH1],  
           V1 );
```

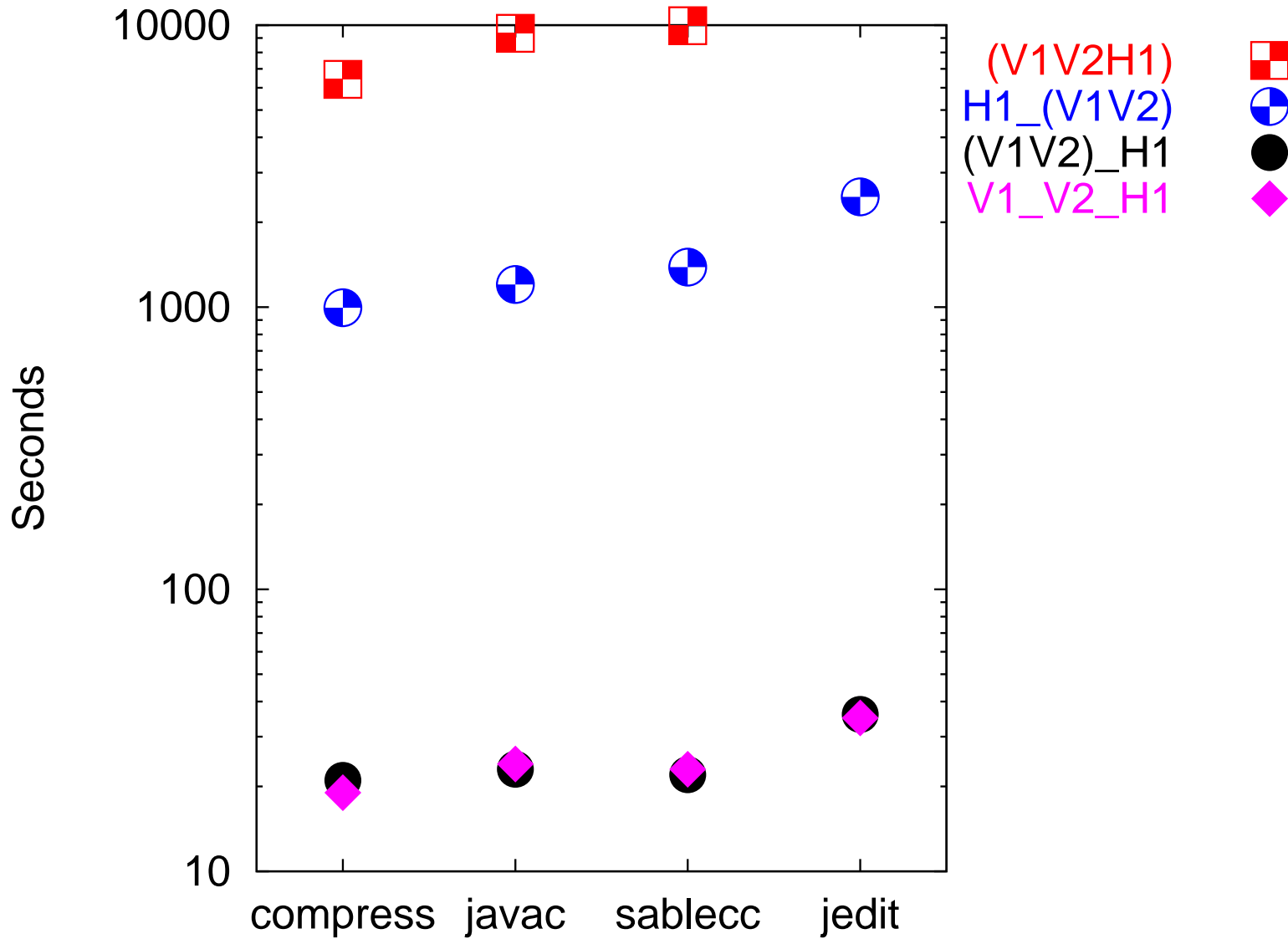
```
newPt2: [V1xH1] =  
  replace( newPt1: [V2xH1],  
           V2ToV1 );
```

V1, V2, H1 make a difference; H2, FD do not.

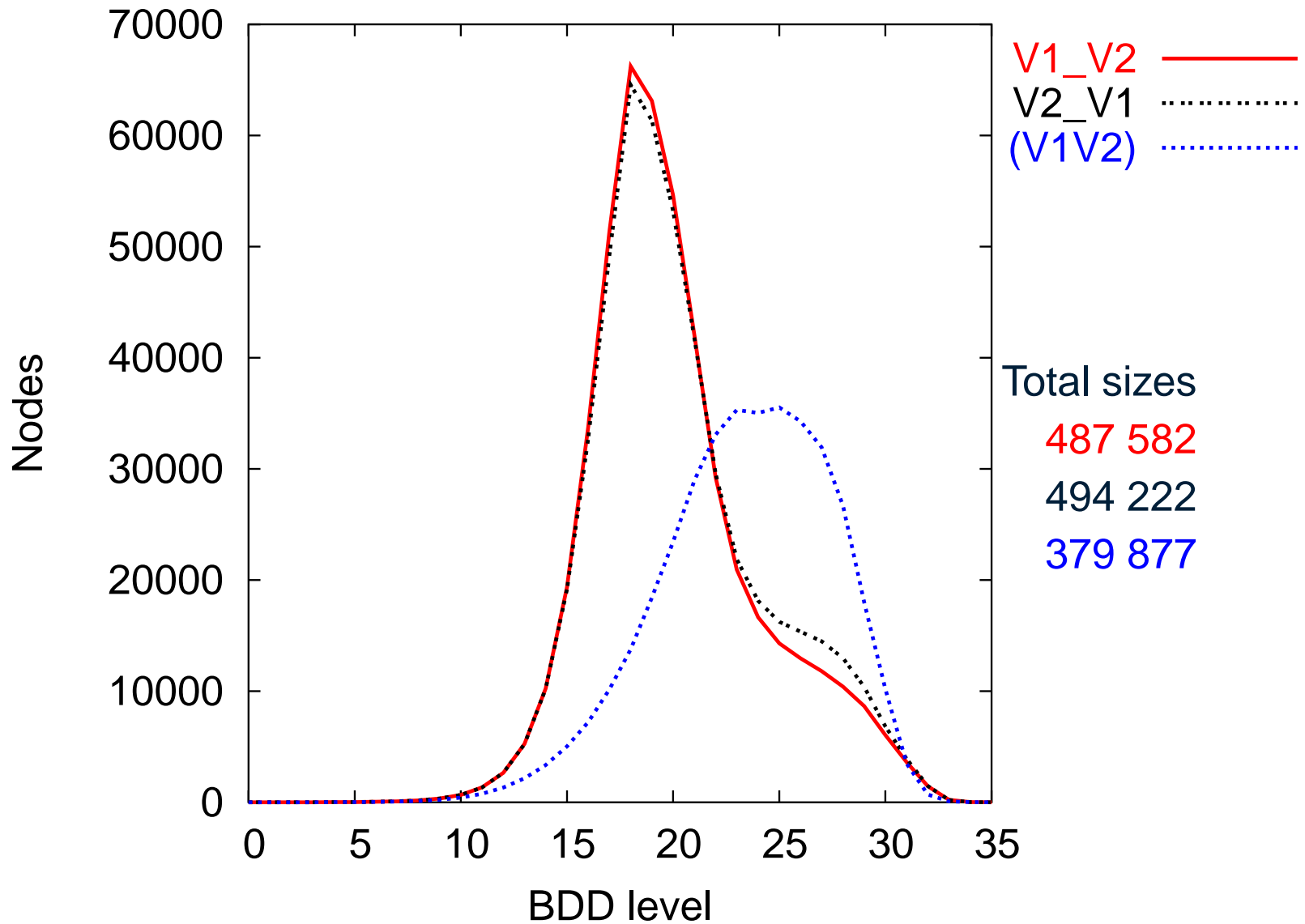
How to find a good ordering?

- Idea:
 - H1 represents points-to sets (large, regular)
 - Put it at the end \Rightarrow big speedup!
- What about V1 and V2?
 - Interleaving them is usually a bit faster than one before the other

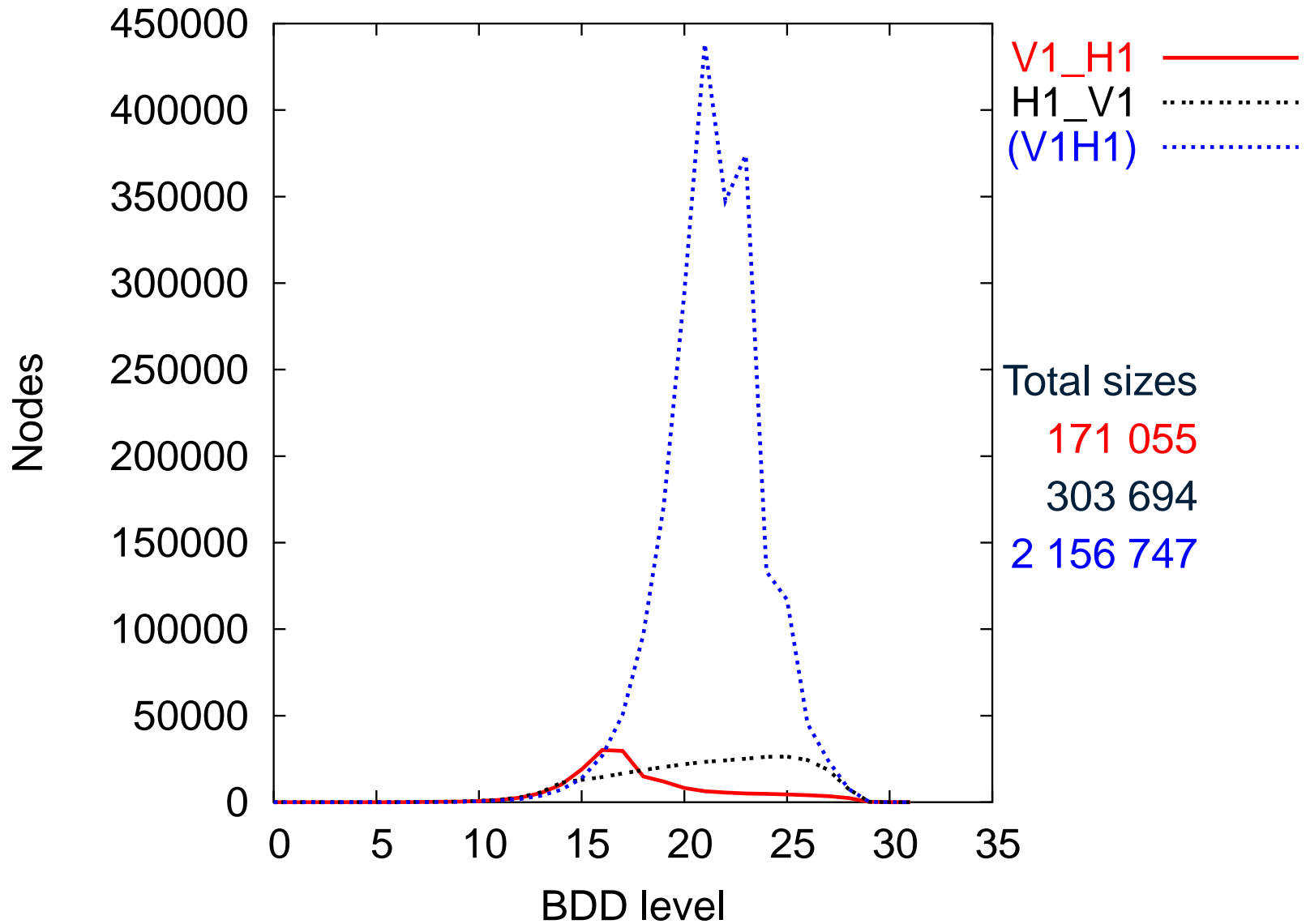
Performance of different orderings



Effect of ordering on *edgeSet*



Effect of ordering on $pointsTo$



Incrementalization

- All sets are re-propagated in each iteration
- Could we propagate only the new elements of each set?
- We found this to work well for Spark
- How well would it work in BDDs?

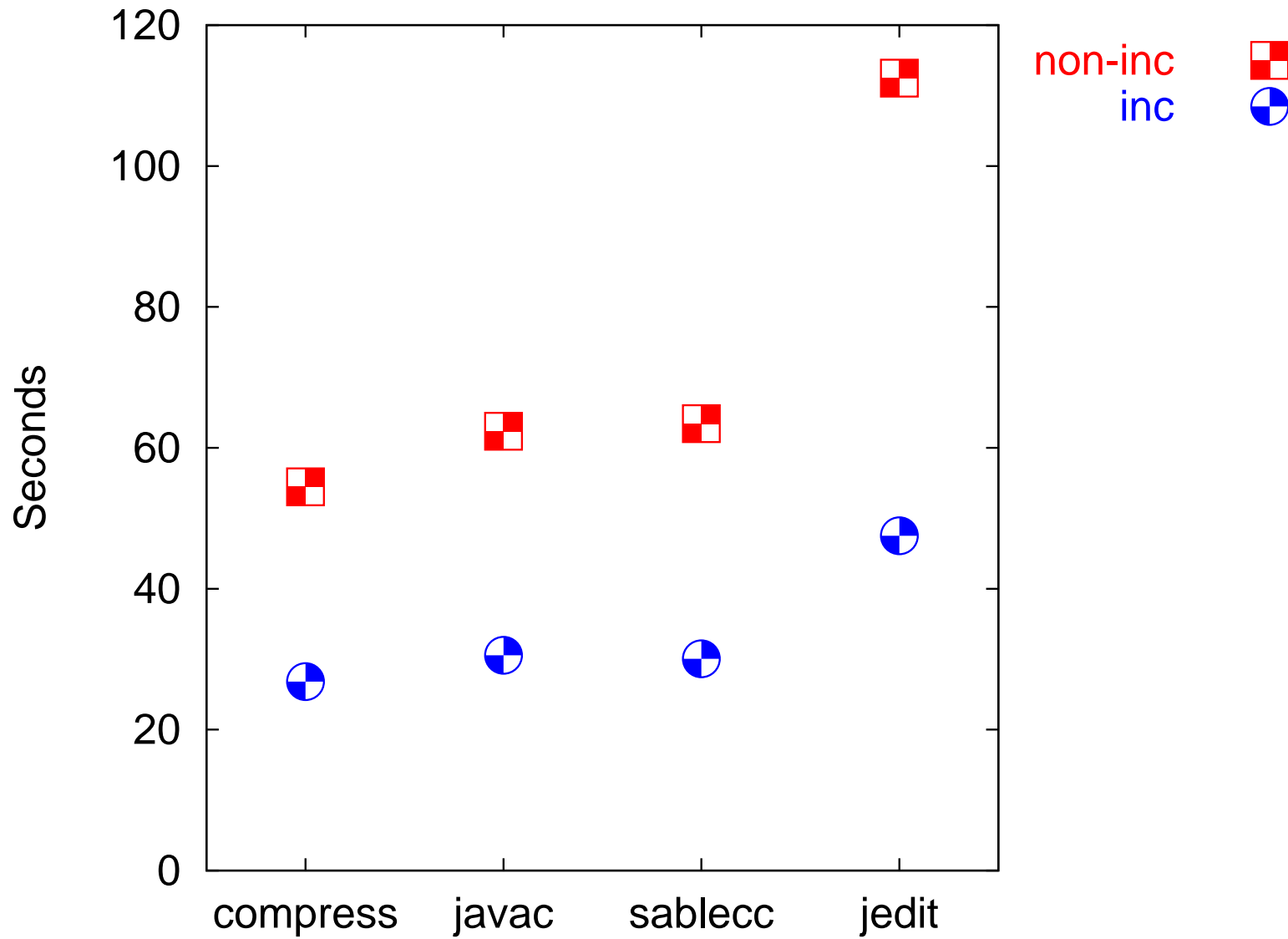
Incrementalization

```
newPt1: [V2xH1] =  
    relprod( edgeSet: [V1xV2],  
             pointsTo:[V1xH1],  
             V1 );  
  
newPt2: [V1xH1] =  
    replace( newPt1: [V2xH1],  
             V2ToV1 );  
  
pointsTo:[V1xH1] =  
    union( pointsTo:[V1xH1],  
           newPt2: [V1xH1] );
```


Incrementalization

```
newPt1: [V2xH1] =  
  relprod( edgeSet: [V1xV2],  
           newPoint: [V1xH1],  
           v1 ) ;  
  
newPt2: [V1xH1] =  
  replace( newPt1: [V2xH1],  
          v2ToV1 ) ;  
  
newPoint: [V1xH1] =  
  setminus( newPt2: [V1xH1],  
           pointsTo: [V1xH1] ) ;  
  
pointsTo: [V1xH1] =  
  union( pointsTo: [V1xH1],  
        newPoint: [V1xH1] ) ;
```

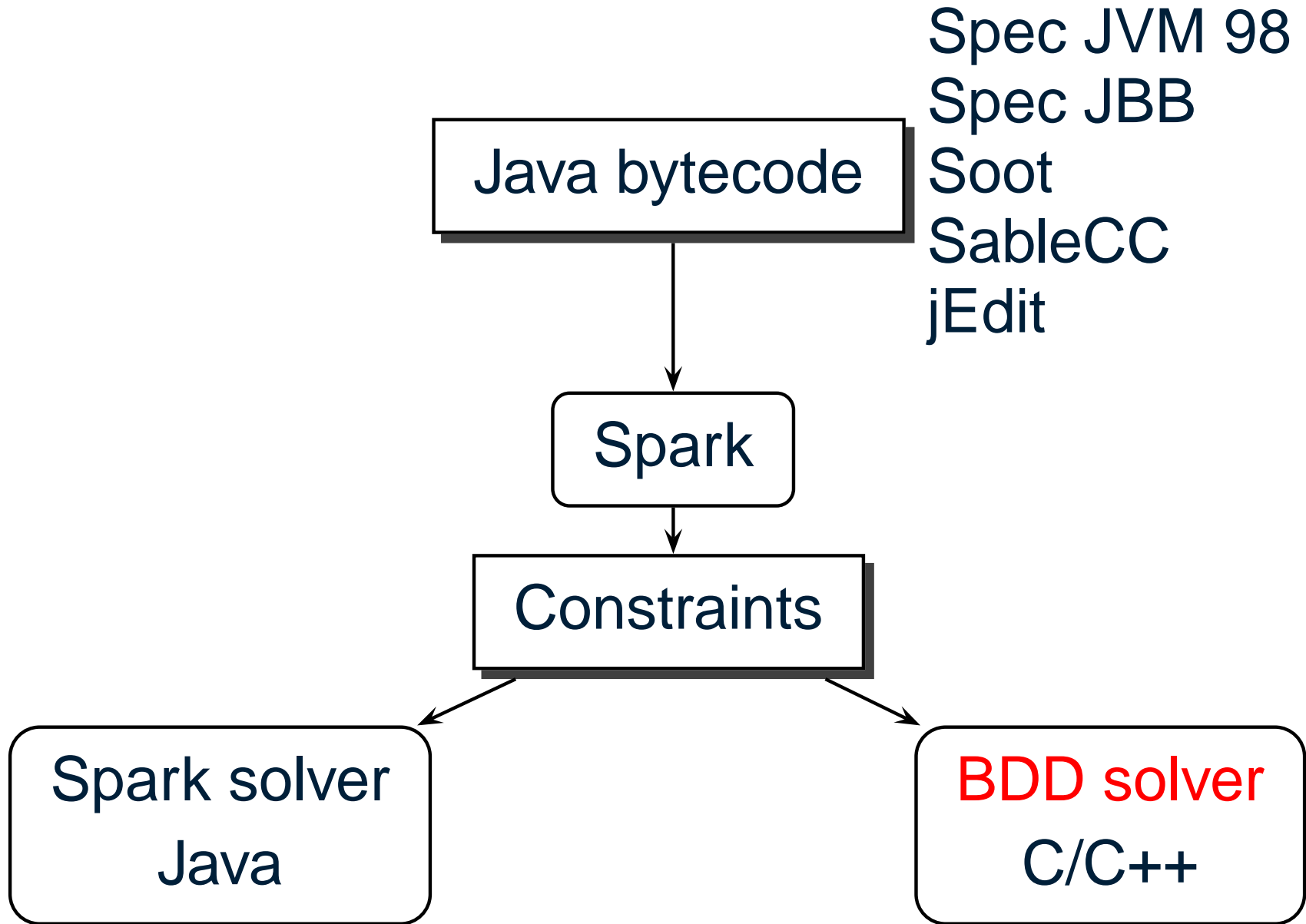
Incrementalization



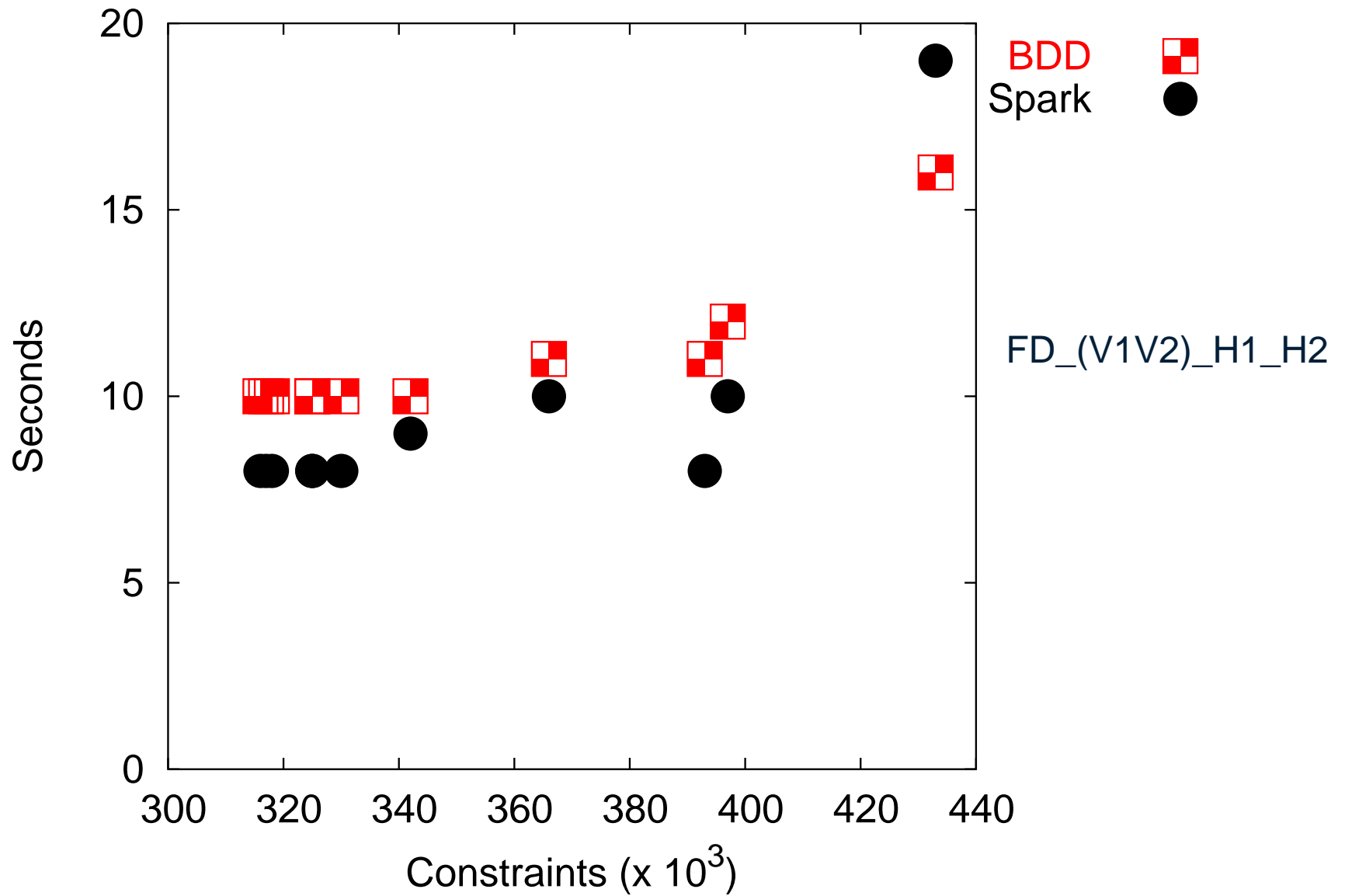
Talk Outline

- Introduction
 - Points-to analysis
 - BDDs
- BDD-PTA algorithm
- Performance tuning
 - Bit ordering
 - Incrementalization
- Overall performance
- Conclusions and future work

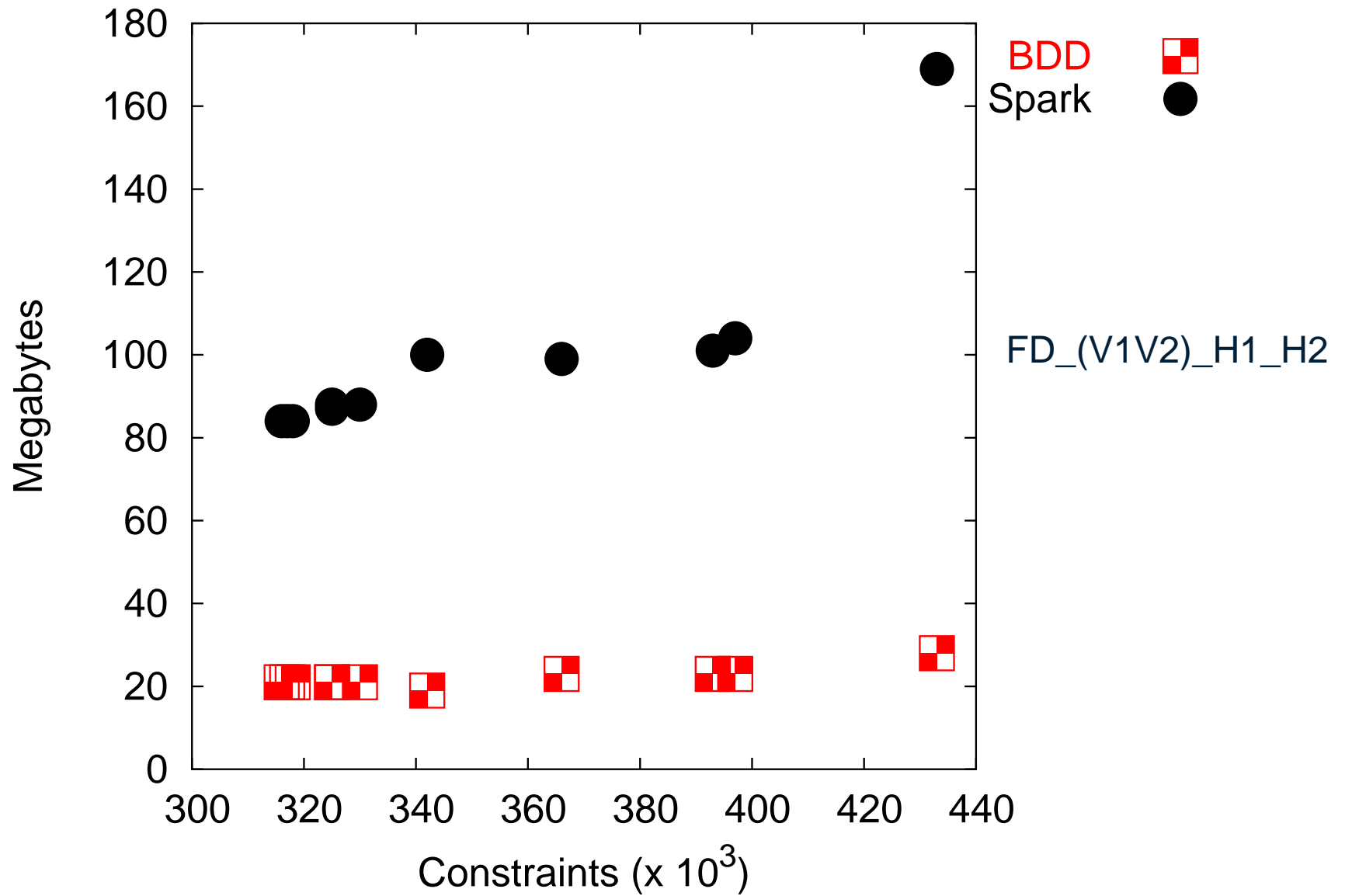
Experiment setup



Overall performance (time)



Overall performance (space)



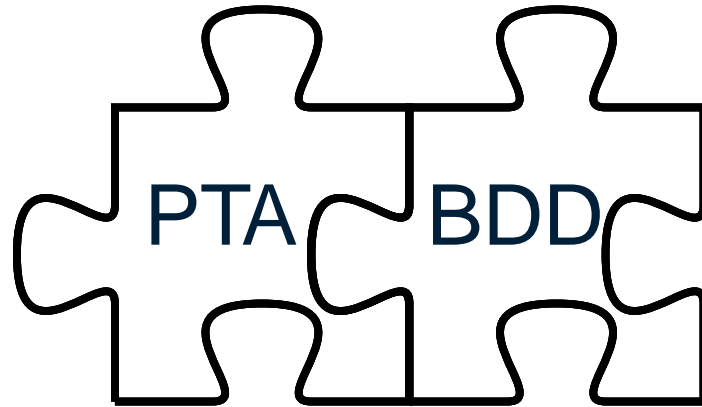
Solving without declared types

- In Java, use declared types of variables to keep points-to sets small
- Without declared types, large sets, traditional solvers do not scale
- May not have declared types (IR does not support them; language dynamically typed)
- Surprisingly, BDD-based solver scales well even without declared types

eg. javac:

| | Set size | BDD size |
|------------|----------|----------|
| with types | 21M | 31MB |
| no types | 366M | 40MB |

Conclusions



- BDDs are a good fit for points-to analysis
- BDDs give reasonably efficient solvers with relatively little effort
- BDDs make it easy to experiment with variations of set-based problems
- Bit ordering is crucial (and we found a good one for points-to analysis)

Future Work

- More heuristics for BDD program analysis
- Library for program analysis using BDDs
- Variations on the points-to analysis
 - Context-sensitivity
- Compute other whole-program information
 - Call graph
 - Interprocedural side-effect analysis
 - ... (suggestions?)