A LINEAR MULTIFACTOR MODEL FOR REITS SELECTION USING GRADIENT MAXIMIZATION

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Abstract

Investing in real estate can provide a good source of diversification as well as a hedge against inflation. In this report, we describe a quantitative ranking model for portfolio construction developed at Desjardins Global Asset Management for the market of Real Estate Investment Trusts (REITs). The linear multifactor model uses eight signals and a gradient maximization algorithm to compute the weights associated to each factor.

We used Java for developing the ranking system and implement the gradient maximization algorithm. We measured the performance of the model by backtesting it over data ranging from 1995 to 2006. We simulated two long-only portfolios, the first one composed of the highest ranked REITs by the model and the second one of the lowest ranked. We restricted trading volume to 10% of a REIT's daily volume and included a \$0.05/share transaction fee.

The simulations results were compared with the MSCI REIT index. The portfolio with the highest ranked REITs outperformed the benchmark significantly by 7.5% on average annually while the one with the lowest ranked REITs underperformed the index by 2.1%. Annual volatility of the returns was higher for both portfolios (16.0% and 16.6%) than for the benchmark (14.6%), but this increase is relatively small.

Acknowledgments

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Finally, my last thoughts go to Cécile, who is always there to listen and advise when needed, and whose love and support help me go through good and difficult times.

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Chapter 1
Introduction

1.1 What Are REITs

A Real Estate Investment Trust (REIT) is a corporation or trust that owns, manages, acquires, develops, and finances income-producing real estate. As a publicly traded company, a REIT allows smaller investors to invest in commercial real estate by purchasing shares of the REIT on a public stock exchange [REI04], and improves liquidity over investing directly in real estate. Various characteristics of REITs include that a REIT must pay 90% of its taxable income through dividend payments, 75% of its assets must be invested in real estate or mortgage loans, and 75% of gross income must be derived from rents, interests on mortgages from real estate property, or gains on sales of real estate assets. Also, a REIT must have at least one hundred shareholders with the five largest owning less than 50% of the total number of outstanding shares.

REITs have grown dramatically as an asset class over the last decade as can be seen from Figure 1.1 which shows the daily transaction volume (in millions) for the NAREIT index. Figure 1.2 provides a break-down of the sector capitalization within the NAREIT index as of September 2005.

Figure 1.1: Daily Dollar Volume (millions) of NAREIT Index

Sector	Number of REITs	Market capitalization (millions)	Weight (%)
Industrial/Office	39	87,401	26.2
Retail	33	83,175	25.0
Residential	27	48,875	14.7
Diversified	18	25,181	7.6
Lodging	19	17,711	5.3
Storage	5	13,259	4.0
Health care	13	16,491	4.9
Specialty	7	14,989	4.5
Mortgage	37	26,273	7.9
Total	**198**	**333,355**	**100**

Figure 1.2: Sector Capitalization of NAREIT Index

1.2 Why Invest in REITs

An advantage of including REITs in a portfolio is that it can reduce overall portfolio risk through diversification since real estate has had in the past a low correlation with other asset classes. Figure 1.3 shows the correlation of the NAREIT index with short-term and long-term bonds as well as large-cap and small-cap indices (Russell 1000 and Russell 2000).

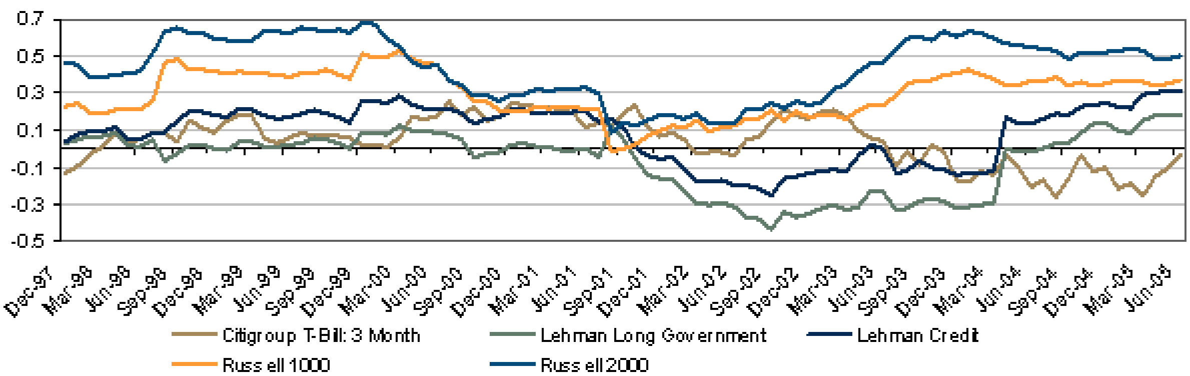

Figure 1.3: REITs Correlation with Various Asset Classes

Another advantage of investing in REITs is that it can provide a hedge against inflation as rental increases provide protection from increase in prices, and delivering strong cash flows regularly through dividend payments [FSH05]. We see in Figure 1.4 that the REITs dividend yield has been higher than the 10-year Treasury bond yield since 1999.

Over the past 7 years, REITs produced a higher total return than the major stock market indices as well as long-term bonds while exhibiting smaller volatility than the Russell indices (see Figures 1.5 and 1.6).

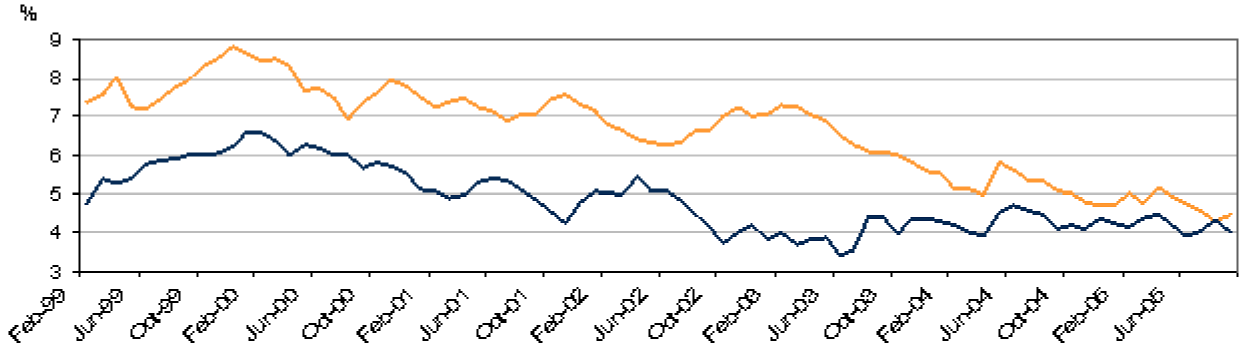

Figure 1.4: REITs Dividend Yield vs. 10-Year Treasury Bonds Yield

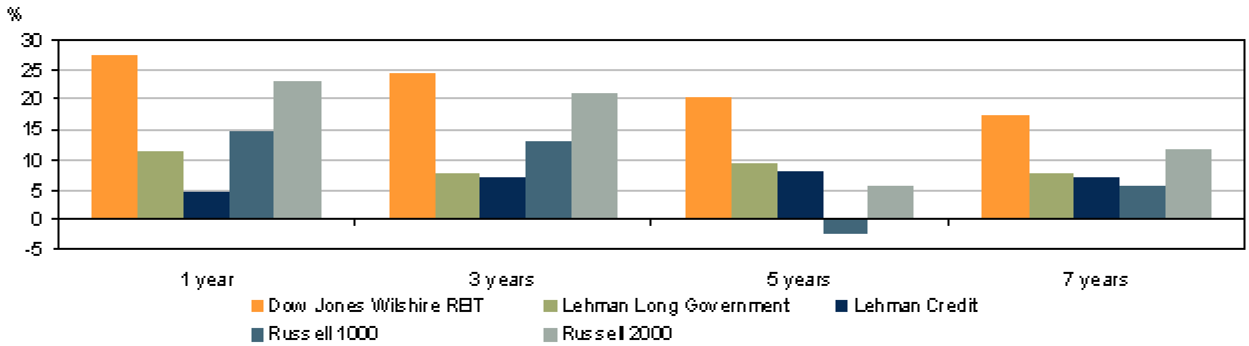

Figure 1.5: Annual Return of Various Asset Classes

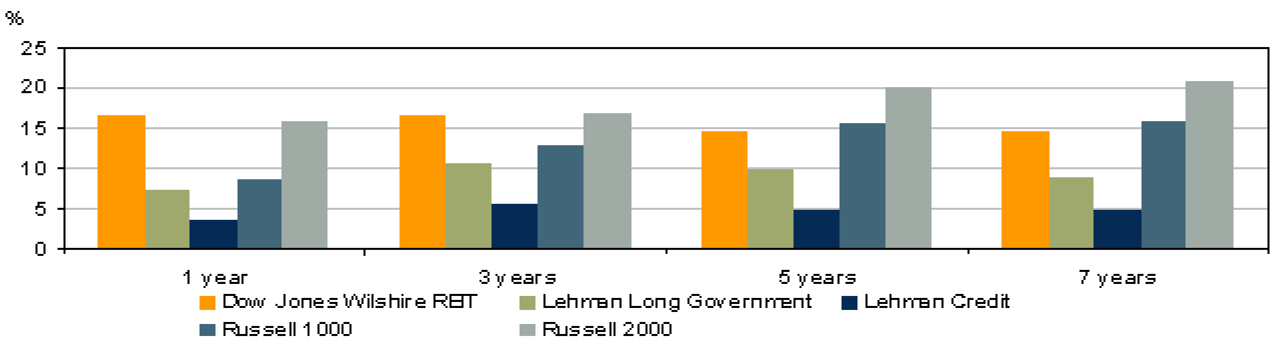

Figure 1.6: Annual Volatility of Various Asset Classes

1.3 A Quantitative System for REITs Selection

The benefits of investing in real estate have led the Quantitative Group at Desjardins Global Asset Management (DGAM) to pursue a project of developing a quantitative system for REITs selection as part of a global portfolio.

We developed a ranking system for REITs based on a multifactor model. The model ranks each REIT by using a linear combination of eight factors. A gradient maximization algorithm is used to compute the weights for the model. We also implemented various optimizations for the model parameters such as the number of REITs to carry in the portfolio and the maximum holding rank for a REIT. The latter provides a rank threshold under which a REIT is sold. We performed various experiments to validate and measure the performance of the model over data ranging from 1995 to 2006. While taking into consideration transaction costs and liquidity, we simulated long-only portfolios composed of the highest ranked and of the lowest ranked REITs by the model. The backtests results produced for the first portfolio annual excess return averaging 7.5%. For the second simulated portfolio, it underperformed the MSCI REIT index by 2.1% on average annually. A production version of this model is currently in place and used for REITs selection in a real portfolio.

1.4 Report Organization

The remainder of this report is organized as follows. The next chapter describes the REITs ranking system. We first provide an overview of the model, then we discuss the selected factors used by the model, and finally we describe the implementation of the gradient maximization algorithm used to compute the weights associated to each factor. In Chapter 3, we specify the data and the environment in which we conducted the experiments, and give details on the implementation of the simulations. In Chapter 4, we discuss the results obtained and compare them with the MSCI REIT index. We conclude with Chapter 5 and suggest potential areas for future work.

Chapter 2
Ranking Model

2.1 Overview

The ranking system is based on a multifactor model where each REIT is assigned a score based on a combination of several factors. The scores are sorted in decreasing order, giving a ranking from the strongest REITs to the weakest ones according to the model. The score of a REIT is a function of the selected factors or signals. For each factor, its value is computed for every REIT. The values are then sorted in decreasing order and divided into deciles. REITs in the top decile get a score of 10 for that particular factor, those in the second best decile have a value of 9, and so on until the bottom decile where REITs are assigned a score of 1. If some data is missing and as a result the value of a factor cannot be computed for a REIT, we assign it a score of 3. For example, if we consider the dividend yield factor, the top 10% of REITs with the highest dividend yield will have a score of 10 while the bottom 10% will get a score of 1. If for any reason we cannot compute the dividend yield of a REIT, we will assign it a value of 3. Finally, the ranking score of a REIT is a linear function of the individual factors scores. The following equation gives the score of REIT r_j where w_i denotes the weight associated with factor f_i.

$$score(r_j) = \sum_{i=1}^{n} w_i f_i(r_j) \tag{2.1}$$

The weights w_i are computed using a gradient maximization algorithm which we describe in Section 2.3. We note that since the weights sum to 1 and that the score for each factor is in the range of 1 to 10, the minimum and maximum ranking scores are 1 and 10 respectively for a REIT.

2.2 Factors

For the n-factor ranking model, we selected eight market factors that can be grouped in three categories: technical, valuation (accounting ratios) and fundamental (analysts recommendations). We used two technical signals: 6-month price momentum and 5-day price contrarian. We used four standard accounting ratios: dividend yield, price-to-earnings, price-to-sales and return on capital. Finally, we used two analysts signals: current recommendation (buy, hold or sell) and recommendation change from the previous month (upgrade, downgrade or no change) from Green Street Advisors, a research and brokerage firm dedicated to the REITs' market. Recommendations from Green Street are made based on their proprietary calculation of net asset value (NAV) as a relative valuation measure for REITs in their respective sector. We note that for the analysts signals, we cannot split them into deciles since there are only three possible values for the current recommendation and recommendation change factors. We thus assign a value of 10, 5 and 1 respectively for buy, hold and sell recommendations. We do the same for upgrade, no change and downgrade for the recommendation change factor. In Figure 2.1, we show a high-level overview of the model.

Below, we provide details of the calculation of the technical signals and accounting ratios used by the model.

- Price momentum: $(P_{t-1} + Div_{t-7,t-1})/P_{t-7}$ where P_{t-1} is the price 1-month prior to time t and $Div_{t-7,t-1}$ is the total dividend from month $t-7$ to month $t-1$. Intermediate term positive momentum is viewed as favorable.

- Price contrarian: $(P_{t-1} + Div_{t-6,t-1})/P_{t-6}$ where P_{t-1} is the price 1-day prior and $Div_{t-6,t-1}$ is the dividend from day $t-6$ to $t-1$. Short-term reversal in the trend is viewed as favorable.

- Dividend Yield: annual indicated dividend / current price.

- Price-to-Earnings: current price / previous twelve months earnings per share.

- Price-to-Sales: current price / previous twelve months sales per share. We consider rents as sales for REITs.

- Return on Capital: annual operating income / net operating assets. Here, assets are real estate owned.

In the ranking model, each factor has a weight associated with it. We describe in the next section the algorithm used to compute these weights.

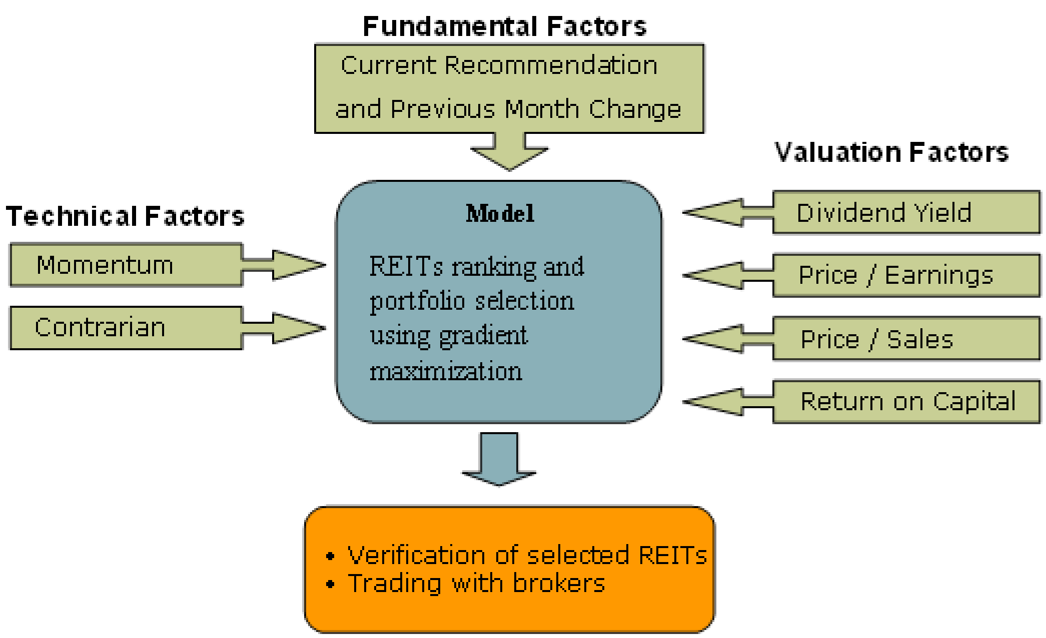

Figure 2.1: Ranking Model for Portfolio Selection

2.3 Gradient Maximization

2.3.1 Algorithm

The gradient maximization algorithm maximizes a function by an iterative process. The function of interest has several variables for which we want to find the values that maximize it. To do so, we first initialize the variables and compute the gradient of the function. The gradient provides the direction of change of each variable that increases the function value. We then modify the values of the variables in the direction given by the gradient and calculate the function value. If an improvement is found, we compute the gradient again and continue this process until we get no more increase in the function value. At this point, we reached a local maximum. Since we cannot find and be certain of a global maximum in financial data, we repeat this entire process with several different initialization of the variables and compare the maximums obtained.

For the REITs model, the function that the gradient maximization algorithm evaluates is a combination of total return and Sortino ratio of a simulated portfolio over a three year period. The variables of the function that we want to estimate are the factors weights.

The Sortino ratio is similar to the Information ratio, which is the ratio of excess return (portfolio return minus benchmark return) divided by the volatility of this excess return. It is defined as:

$$\text{Information Ratio} = \frac{E[R_p - R_b]}{\sigma}$$

where R_p and R_b are the monthly portfolio and benchmark returns, $E[R_p - R_b]$ is the expected monthly excess return and calculated as the average of a serie of monthly excess returns, and $\sigma = \sqrt{Var[R_p - R_b]}$ is the standard deviation of a serie of monthly excess returns. The difference between the Sortino and the Information ratio is that in the Sortino case, σ is replaced by σ_d which is the downside volatility where only negative excess returns are used in the standard deviation calculation. We have chosen this ratio over the more standard Information ratio since we are not concerned with the volatility of positive excess returns.

The gradient maximization algorithm has been implemented as described by Brush and Schock [BS95]. We highlight the six steps in this process below:

(I) Initialize weights randomly.

(II) Simulate portfolios composed of the highest ranked REITs for all combinations of weights adjusted by +/- 20% over three years of daily data.

(III) Compute total return and Sortino ratio for each simulation.

(IV) Modify the weights in the direction given by the best simulated portfolio in Step III, and perform simulations in this direction until a local maximum is reached.

(V) Repeat Steps (II), (III) and (IV) until no improvement is found.

(VI) Repeat the entire process from Step (I) with different initial weights.

It should be noted that the gradient computation is done by Steps II and III, i.e. we adjust the weights in every direction, we simulate portfolios using these weights, and we compute portfolio return and Sortino ratio. The direction of weights adjustments (the gradient) can then be found by choosing the portfolio that best combines total return and Sortino ratio (the criteria to maximize).

2.3.2 Example

As a concrete example, we consider a 2-factor model and initialize each of the two weights to 0.5. We then modify the weights by +/- 20% and simulate portfolios of REITs. We compute the total return and the Sortino ratio for all portfolios. We assign a score to each by combining these two measures. Different functions can be used for this purpose. We use the following equation:

$$score(p_i) = \frac{r_i}{\max_j r_j} + \frac{s_i}{\max_j s_j} \tag{2.2}$$

where p_i is the i$^{\text{th}}$ portfolio simulation, and r_i and s_i are the total return and Sortino ratio for portfolio i.

Portfolio	w1	w2	Total return (r)	Sortino ratio (s)	r / max(r) + s / max (s)
1	0.4	0.4	31%	0.65	1.66
2	0.4	0.5	39%	0.71	1.95
3	0.4	0.6	37%	0.70	1.88
4	0.5	0.4	35%	0.68	1.79
5	0.5	0.5	35%	0.67	1.79
6	0.5	0.6	36%	0.71	1.87
7	**0.6**	**0.4**	**38%**	**0.75**	**1.97**
8	0.6	0.5	34%	0.68	1.78
9	0.6	0.6	33%	0.68	1.75

Figure 2.2: Example of Determining the Gradient

In Figure 2.2, we show an example of portfolios with weights adjusted by +/- 20% from the initial weights both equal to 0.5 along with possible simulations results. We notice that the highest total return and Sortino ratio are 39% (portfolio 2) and 0.75 (portfolio 7) respectively, each coming from a different portfolio simulation. The last column shows the score assigned to each portfolio simulation using equation 2.2. The best one is Portfolio 7 (shown in bold) with values for w_1 and w_2 of 0.6 and 0.4, and a score of 1.97. We note here that among all portfolios, this one has the second highest total return but the highest Sortino ratio. Thus, the direction of the gradient for weight w_1 is positive (since 0.6 is higher than the initial 0.5 value) and for w_2 it is negative (since 0.4 is lower than 0.5).

Figure 2.3 (from [BS95]) shows the gradient maximization iterative process for a 2-factor model (two weights to estimate). Point 1 is the starting point where weights are initialized (Step I). The black dots surrounding Point 1 in the figure correspond to the modification of the weights (Step II) to find the direction of highest improvement (Step III). We see in the picture that this direction is pointing towards Point 2. Weights are adjusted in this direction (Step IV) until a local maximum is reached (Point 2). Steps II, III and IV are performed until Point 3 is reached, and again until Point 4. At that point, modifying the weights does not improve the solution.

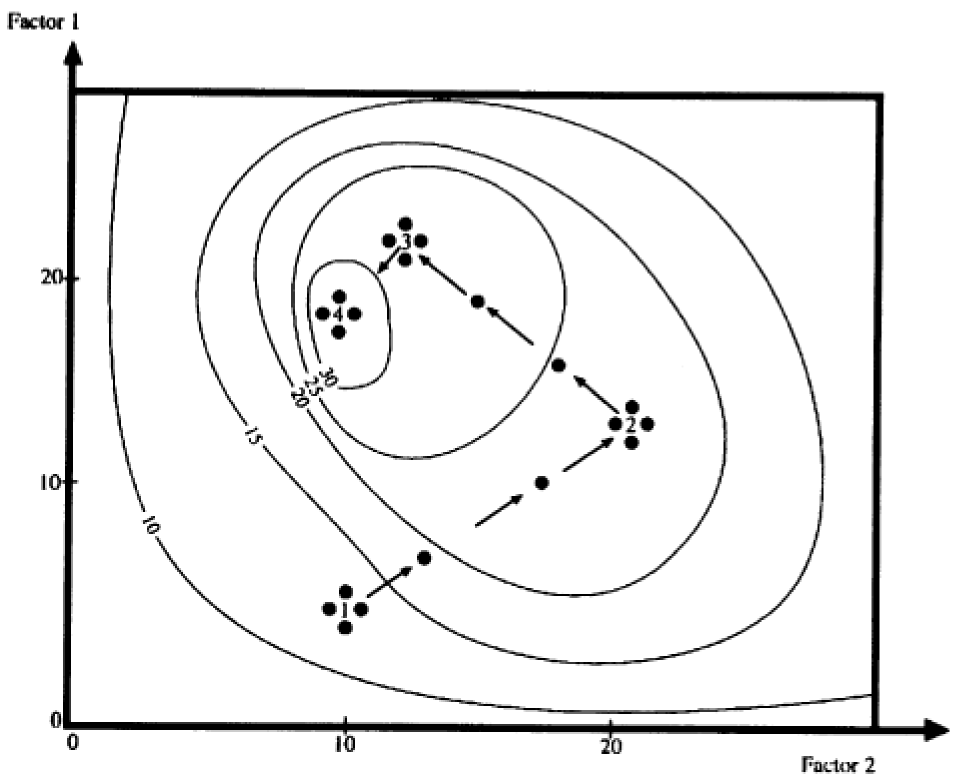

Figure 2.3: Illustration of the Gradient Maximization with Two Factors

2.4 Summary

In summary, the ranking model is a 4-step quantitative process:

(I) For each of the eight factors, rank REITs based on the values of the factor.

(II) For every factor, divide the ranking into deciles and assign them a score of 1 (bottom decile) to 10 (top decile).

(III) Compute weights for each factor using the gradient maximization algorithm described in Section 2.3.

(IV) Compute the total score for each REIT using equation 2.1.

In this chapter, we described the ranking system used for selecting REITs in a portfolio. In the next chapter, we discuss the experimental framework in which we ran simulations to evaluate the performance of the model.

Chapter 3
Experimental Framework

In this chapter, we describe the environment in which we conducted experiments to evaluate the performance of the ranking model. The next section describes the data that we used. Section 3.2 specifies the systems and tools used for the development of the model and for the simulations. Finally, in Section 3.3, we give details of the simulations, including the various parameters used by the model and the implementation of the train/test rolling window employed to evaluate its performance.

3.1 Data

For our experiments, we used data ranging from January 1995 to June 2006 from the S&P Compustat database, including daily prices, dividend data at the ex-date, and quarterly fundamental data to compute price-to-earnings, price-to-sales, dividend yield and return on capital ratios. For the quarterly fundamentals, we used the Point-in-Time package in Compustat which shows the data that was available in the database historically at a given point in time. For example, if a REIT had its earnings per share for the last quarter of 1999 restated in 2001, then in our simulations we used the unrestated value in 2000 but the restated one in 2001. Hence, the Point-in-Time package keeps track of financial restatements making backtests results more reliable.

We considered for our sample the set of REITs covered by Green Street Advisors, including those that do not exist anymore, have merged, or stopped being followed. To limit the effect of survivorship bias, we included every REIT that received monthly recommendations between 1995 and 2006 by Green Street. This gave us approximately 150 REITs for the entire period, of which about ten were eliminated due to unavailability of prices or quarterly data. We note that typically, for any given month, there were around 60 to 70 REITs covered by Green Street.

3.2 Systems and Tools

The development of the model as well as the simulations have been made in the Java programming language using JDK 1.5.0 Update 6 within the Cygwin environment. The machine used for the implementation and for running the experiments was an Intel Pentium 4, 2.8 GHz system with 2 GBs of RAM running Microsoft Windows XP Professional Version 2002 Service Pack 2.

3.3 Simulations Implementation

3.3.1 Parameters

In Section 2.3, we described how the gradient maximization algorithm computes the weights associated to each factor. In our simulations, two additional parameters, which we called *PF_SIZE* and *MAX_HOLD_RANK*, also needed to be optimized. The first one specified the number of REITs to buy when there was cash available in the portfolio. For example, if after optimization *PF_SIZE* was set to 12, then every day if the portfolio had money available then the top 12 ranked REITs by the model were purchased with equal weights. For the second parameter, it provided the maximum rank of a REIT to stay in the portfolio. For example, if the maximum rank was 30, then whenever the rank of a REIT contained in the portfolio fell below 30, we got rid of it. We added these two parameters to the factors weights to be optimized as part of the gradient maximization algorithm.

In our simulations, we allocated an initial capital of 10 millions dollars for the portfolio. The simulations were performed daily and whenever there was money available in the portfolio, the highest *PF_SIZE* ranked REITs by the model were purchased (equal-weighted).

If a REIT held in the portfolio had a rank lower than *MAX_HOLD_RANK*, we sold it. When establishing a position for a REIT, we limited the maximum number of shares that we bought or sold to 10% of the REIT's daily volume. We included a $0.05 fee for the cost of each share transaction, and used the average of the daily high, low and close price for the transaction price.

3.3.2 Rolling Window

We implemented the backtesting of the ranking system by using the rolling train/test window model typically used in learning algorithms and described in [Lev95]. We performed sequential validation of the model by training the gradient maximization over three years of continuous data, and then evaluated the computed weights on the subsequent six months of data. The gradient maximization algorithm was thus invoked every six months. Since our data is from January 1995 to June 2006, the simulations of the portfolios started in January 1998 using computed weights by the gradient maximization on data from January 1995 until December 1997. The performance of the portfolios was then measured using these weights on six months of out-of-sample data from January 1998 to June 1998. In July 1998, the gradient maximization algorithm was invoked again, and the data for training and testing were moved accordingly, i.e. the weights were recomputed using data from July 1995 until June 1998 and the performance was evaluated using the new computed weights on out-of-sample data from July 1998 to December 1998. The window of data for training and testing was moved forward this way until the end of the simulations in June 2006. We note that the signals were calculated daily. Hence, the model ranked REITs on a daily basis, and positions were evaluated and rebalanced accordingly.

We simulated two long-only portfolios: the first one composed of the highest ranked REITs by the model and the second one of the lowest. We used the gradient maximization algorithm to compute weights for each. Invoking the gradient maximization for selecting low performing REITs is the opposite of for high performing REITs, i.e. instead of maximizing total return and Sortino ratio we minimize it. In Appendix A, we show the weights computed every six months for both simulations.

Chapter 4
Simulations Results

In the previous chapter, we discussed the experimental framework and the implementation of the highest- and lowest-ranked REITs simulations using the gradient maximization algorithm. In the following sections, we present the performance measurements of the simulated portfolios.

4.1 Measurements

The performance measurements of the ranking model start in January 1998 and end in June 2006. We list and provide details of the various measurements for the simulations results contained in Tables 4.1 and 4.2.

- Position Size (μ): daily average number of REITs held in portfolio.
- Position Cash (μ and σ): daily average and standard deviation of cash position as a percentage of total portfolio value.
- Monthly Turnover (μ and σ): A measurement of portfolio trading activity. A high turnover value means assets are bought and sold frequently and thus high transaction costs are incurred. The value for a given month is calculated by dividing the monthly total dollar value of purchases or sales (whichever is less) by the average total value of the portfolio in the month.

- Monthly Return (μ): average monthly return of portfolio.
- Yearly Return: portfolio return in a given year.
- Total Return: portfolio return from the simulation start date until the end date.
- Annual Volatility: standard deviation of the serie of portfolio monthly returns multiplied by $\sqrt{12}$.
- Maximum Drawdown: largest percentage loss from a peak in portfolio value to a bottom.
- Recovery Date: date at which the portfolio value recovered from the maximum drawdown.
- Excess Return (μ and σ): average and standard deviation of the serie of monthly excess returns over the MSCI REIT index. Excess return for a given month is calculated as the portfolio return for that month minus the index return.
- Months Positive: number of months where the excess return is positive.
- Tracking Error: standard deviation of the serie of monthly excess returns multiplied by $\sqrt{12}$.
- Information Ratio: average of the serie of monthly excess returns divided by the standard deviation of the serie.
- Sortino Ratio: same as the Information ratio except that only negative excess returns are taken into account when computing the standard deviation of the serie of excess returns.

4.2 Results

The results of the ranking model simulations are shown in Table 4.1. It should be noted that the yearly return for 2006 only includes the first six months. The first row of Table 4.1

shows that the average number of REITs held daily in the highest- and lowest-ranked portfolios is about the same. Similarly for the daily cash position for both models which had only 0.1% on average, and hence our restriction of limiting transaction volume to 10% of the daily volume did not cause liquidity issues. For both strategies, the turnover was around 31%, a large but not excessive value. This means that each portfolio was rolled over approximately 3 1/2 times per year. The monthly return for the highest-ranked portfolio was 1.68%, much higher than for the benchmark (1.07%). For the lowest-ranked simulation, the monthly return was 0.91%, which is below the index. We note that if we wanted to be market neutral, this difference would not be significant enough to short the lowest-ranked portfolio since taking into account the repo rate (the cost of borrowing shares for shorting), the performance would approximately be the same as the index but with more risks (see discussion on volatility below).

In the middle rows, we see the yearly returns from 1998 to 2006 (only the first six months are included in 2006). In every year, the highest-ranked simulation outperformed both the lowest-ranked portfolio and the benchmark. For the lowest-ranked portfolio, it underperformed the benchmark each year except for 2004 and 2006. The highest-ranked portfolio thus outperformed much more the benchmark and more consistently than the lowest-ranked portfolio underperformed it. The total returns for both simulations as well as for the index were 390.9%, 125.0% and 169.6% respectively, with annual volatility of 16.0%, 16.6% and 14.6%. Although the volatility of the benchmark returns was smaller than the ones for the simulations, the difference is not significant compared with the excess returns generated by the highest-ranked portfolio. However, this increase in risks may not justify the shorting of the lowest-ranked portfolio given its small underperformance.

The last two rows of Table 4.1 show the maximum drawdown and recovery date for the simulations. For the first portfolio, the largest decrease in value from any given point was -23.9% while the index dropped by -12.4% for the same period. It then took 2 1/2 months for the portfolio to return to that peak. For the second portfolio, the maximum drawdown was -31.1% (-26.6% for the benchmark) but it took more than seven months to recover from this drop.

Measurement		Portfolio Highest-Ranked	Portfolio Lowest-Ranked	MSCI REIT Index
Position Size	μ	22.7	20.1	-
Position Cash	μ	0.1%	0.1%	-
	σ	0.3%	0.2%	-
Monthly Turnover	μ	30.7%	31.1%	-
	σ	20.4%	23.6%	-
Monthly Return	μ	1.68%	0.91 %	1.07%
Yearly Return	1998	-0.29%	-18.5%	-16.7%
	1999	0.28%	-5.7%	-4.6%
	2000	39.9%	20.9%	26.8%
	2001	19.2%	9.5%	12.8%
	2002	10.3%	-2.7%	3.6%
	2003	45.1%	34.2%	36.7%
	2004	35.1%	33.8%	31.5%
	2005	16.2%	9.4%	12.1%
	2006[a]	17.1%	15.7%	13.5%
Total Return		390.9%	125.0%	169.6%
Annual Volatility		16.0%	16.6%	14.6%
Maximum Drawdown		-23.9%	-31.1%	-12.4% & -26.6%
		2001/08/24 to	1998/01/21 to	-
		2001/09/21	1999/12/14	-
Recovery Date		2001/12/05	2000/07/26	-

[a] from January to June 2006

Table 4.1: Simulations Results

We show cumulative total returns of the simulated portfolios and of the MSCI REIT index (the benchmark) in Figure 4.1 for a $100 investment at the beginning of the period in January 1998 until June 2006. We see in the figure that the highest-ranked strategy outperformed the lowest-ranked portfolio and the benchmark significantly for the entire period. The lowest-ranked simulation underperformed the index modestly, and hence we do not recommend it as a shorting strategy since a cost of 1.0-1.5% annually would be incurred for borrowing shares.

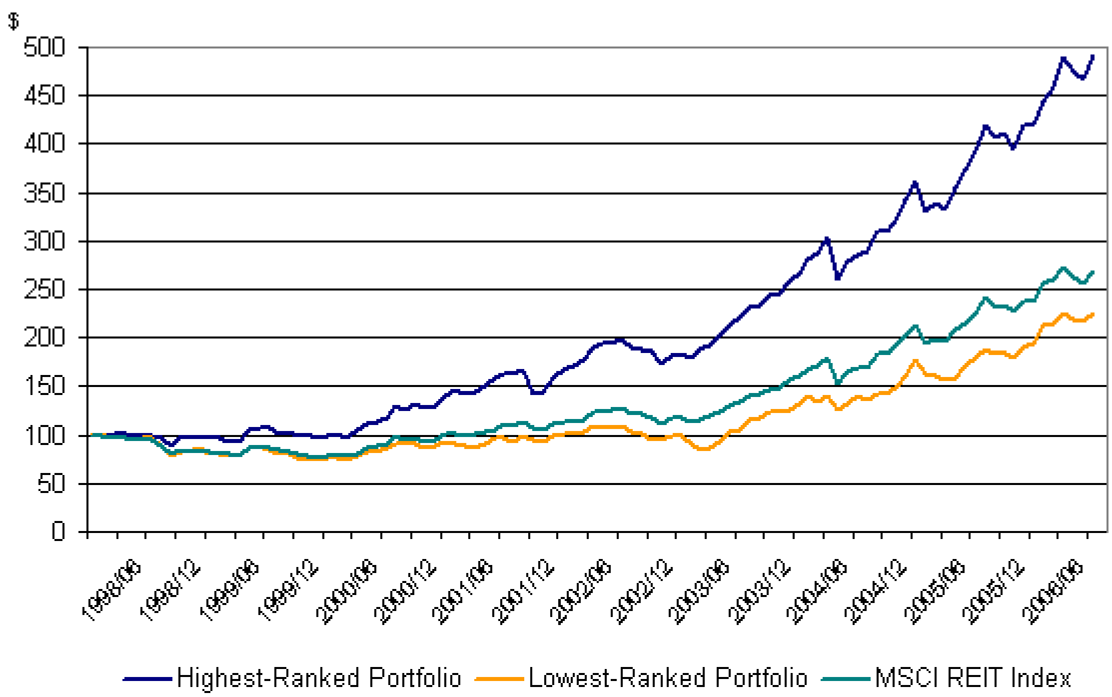

Figure 4.1: Cumulative Returns of Simulated Portfolios and Index

Table 4.2 shows excess return measurements for the highest- and lowest-ranked portfolios over the benchmark. The average monthly excess return for both strategies are 0.62% and -0.15%, with volatility of 1.9% and 2.6%. The results are thus much better for the highest-ranked portfolio since the excess return is higher and the volatility smaller. The second row gives the number of months in which the excess return is positive. In Appendix B, we provide the returns for every month from January 1998 to June 2006 for the simulations as well as for the benchmark. We see that the highest-ranked portfolio outperformed the benchmark in 70% (71/102) of the months whereas the lowest-ranked one underperformed the index 53% (54/102) of the time. The third row shows the tracking error measure (the annualized excess return volatility) for the models. It is better for the highest-ranked portfolio since its value is smaller. Finally, the bottom two rows show the Information and Sortino ratios. The ratios of 1.11 and 1.43 for the first portfolio are very good, but for the second one, the ratios of -0.21 and -0.29 are average.

		Portfolio	
Measurement		Highest-Ranked	Lowest-Ranked
Excess Return	μ	0.62%	-0.15%
(monthly)	σ	1.9%	2.6%
Months Positive		71 / 102	48 / 102
Tracking Error		6.6%	9.0%
Information Ratio		1.11	-0.21
Sortino Ratio		1.43	-0.29

Table 4.2: Excess Return Measurements

The next two figures show the annual excess return and the tracking error of the simulations. In Figure 4.2, we see that the highest-ranked portfolio had a positive excess return in every year. For the lowest-ranked strategy, it underperformed the index in seven of the nine years, 2004 and 2006 being positive. In Figure 4.3, we note that the tracking error for the first simulation is rather small especially in recent years (only in 2001 it is somewhat large). We notice the opposite for the second simulation where the tracking error is high in later years.

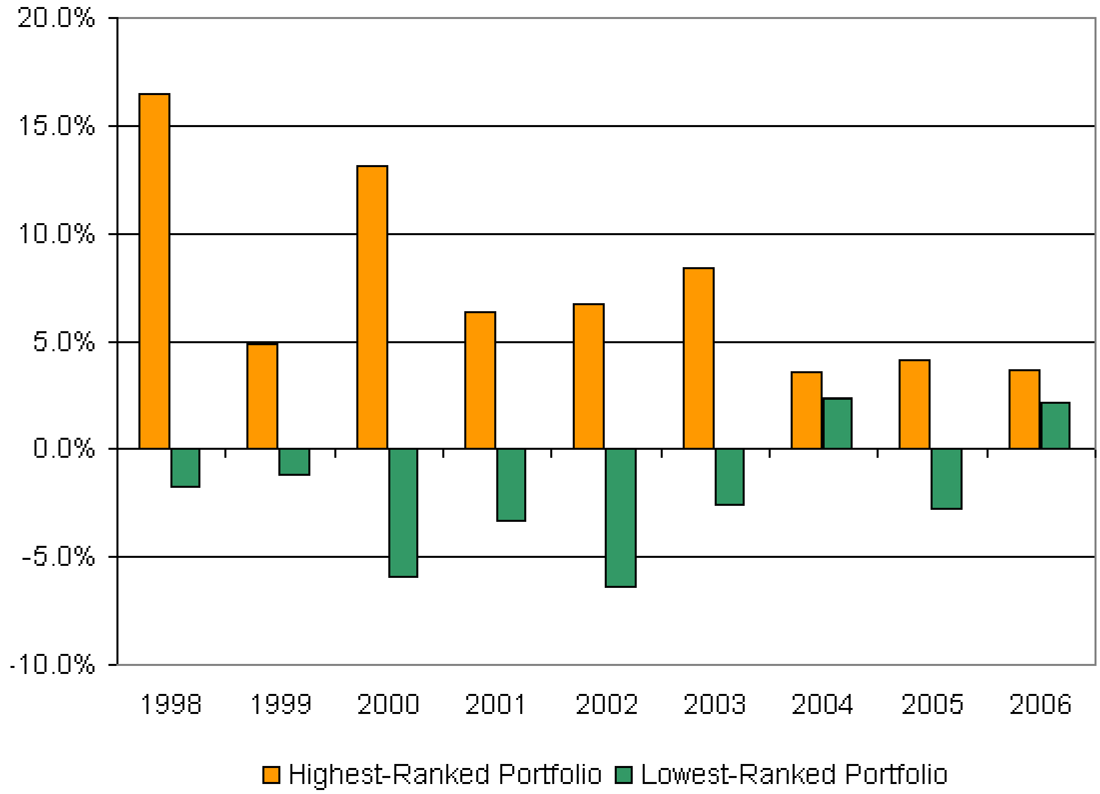

Figure 4.2: Annual Excess Return of Simulated Portfolios over Index

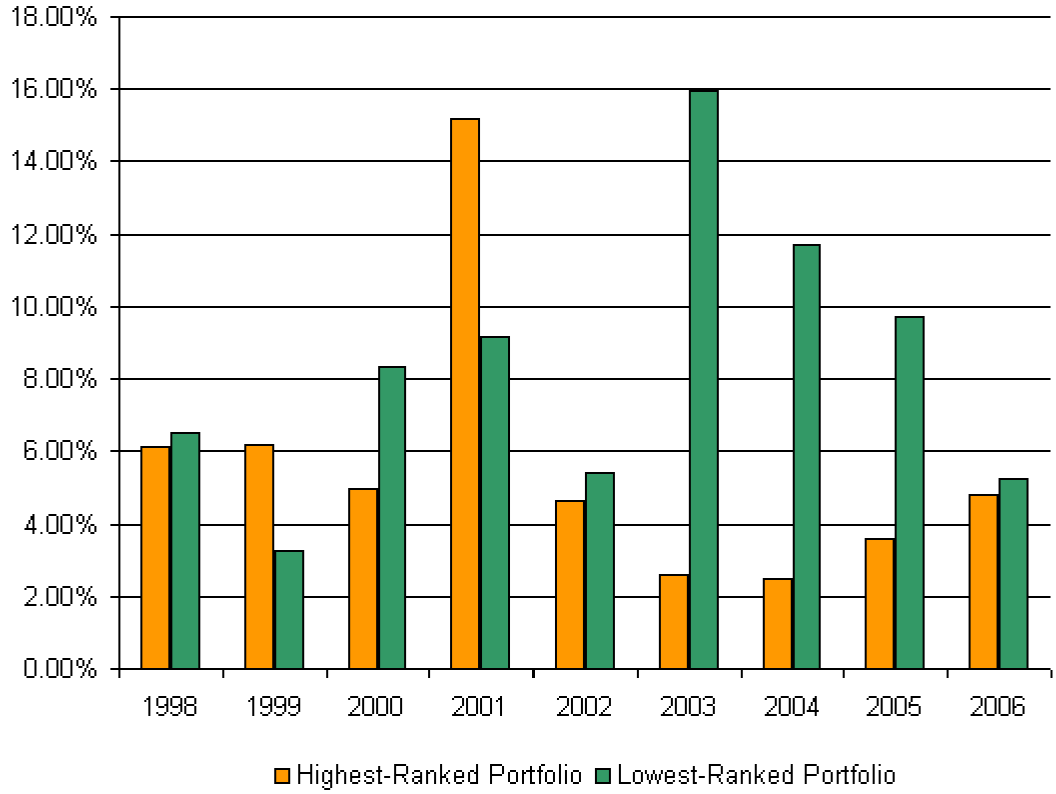

Figure 4.3: Annual Tracking Error of Simulated Portfolios

Chapter 5
Conclusions and Future Work

5.1 Conclusions

In this report, we first gave a description of REITs and discussed several advantages of investing in them. We then described the development of a quantitative system for REITs selection based on a multifactor model. Eight factors were chosen for the model: price momentum, price contrarian, current analyst recommendation, change in recommendation from the previous month, dividend yield, price-to-earnings, price-to-sales and return on capital. The model combined these signals linearly, and used a gradient maximization algorithm to compute the weights associated to each factor.

We conducted experiments by backtesting the model using a rolling train/test window over data ranging from 1995 to 2006. We performed simulations of portfolios composed of the highest- and lowest-ranked REITs by the model. We limited transaction volume to 10% of a REIT's daily volume and assumed a fee of $0.05 for each share transacted.

Finally, we measured the performance of the simulations and compared the results with the MSCI REIT index. The results of the simulations show that the highest-ranked strategy outperformed the benchmark significantly and had returns in excess of 7.5% annually on average. In every year from 1998 to 2006, the excess return was positive while the annual volatility was only 1.4% higher (16.0% versus 14.6% for the benchmark). For the lowest-ranked simulation, the portfolio underperformed the index by 2.1% on average annually with a 2.0% increase in volatility (16.6%).

5.2 Future Work

5.2.1 Rebalancing

In our simulations, when a set of REITs was selected to be purchased, each one of them was allocated the same amount of money, i.e. positions were equal-weighted. In contrast, different trading methods could be explored. An interesting alternative would be the anticor algorithm presented by Borodin, El-Yaniv and Gogan [BEYG04], which on the premise that constant rebalancing can improve performance, have developed a technique that takes advantage of predictable correlation between pairs of stocks.

5.2.2 Classification and Regression Trees

Classification and Regression Trees (CART) is a non-parametric method that classifies observations into categories, or in the continuous case into regression trees. CART is modeled in the form of binary trees that represent decision rules. Each internal node in the binary tree split sample data into two nodes based on the variable value associated with that node (in this case the associated factor's value). As such, CART is also known as binary recursive partitioning since parent nodes are always split into exactly two child nodes and recursive because the process can be repeated by treating each child node as a parent [whies]. This hierarchical representation results in data clusters starting from the root node with the entire learning data and end with small group of homogeneous observations [And05].

It would be interesting to experiment with a CART model since it has been a proven robust data-mining and data-analysis tool used in the past and capable of discovering important patterns and relationships in highly complex data. CART's set of decision rules could also provide intuition in the relationships between the different factors affecting the performance of REITs, and consequently provide a high degree of result's interpretability.

5.2.3 Combining Models

It is known that combining various predictors can improve upon using predictors individually. Thus, the ranking model described in this report could be used in conjunction with a CART model. An investigation of committee and bagging techniques [KV95, Bre94] could be performed to see whether factors affecting REITs performance could be generalized.

Appendix A
Gradient Maximization Computed Weights

In the following two tables, we show the weights that were computed every six months by the gradient maximization algorithm for the highest- and lowest-ranked portfolio simulations. We note that the weights in a row may not always sum to 1 because of rounding to the second decimal. In the tables, the corresponding factors are:

- F1: Contrarian
- F2: Current Green Street Recommendation
- F3: Recommendation Change from Previous Month by Green Street
- F4: Momentum
- F5: Dividend Yield
- F6: Price-to-Earnings
- F7: Price-to-Sales
- F8: Return on Capital

Date	Factor							
	F1	F2	F3	F4	F5	F6	F7	F8
1998/01/01	0.23	0.05	0.3	0.22	0.02	0.03	0.05	0.09
1998/07/01	0.15	0.14	0.19	0.23	0.04	0.04	0.12	0.09
1999/01/01	0.24	0.07	0.35	0.16	0.02	0.05	0.06	0.06
1999/07/01	0.23	0.27	0.29	0.06	0.04	0.02	0.07	0.01
2000/01/01	0.3	0.08	0.24	0.16	0.03	0.05	0.08	0.06
2000/07/01	0.14	0.18	0.25	0.08	0.05	0.06	0.18	0.05
2001/01/01	0.12	0.18	0.25	0.1	0.04	0.06	0.17	0.07
2001/07/01	0.22	0.05	0.37	0.07	0.08	0.09	0.1	0.03
2002/01/01	0.13	0.09	0.15	0.17	0.12	0.13	0.14	0.07
2002/07/01	0.15	0.11	0.15	0.23	0.12	0.08	0.08	0.09
2003/01/01	0.13	0.13	0.21	0.16	0.11	0.08	0.05	0.12
2003/07/01	0.12	0.14	0.13	0.14	0.09	0.14	0.11	0.14
2004/01/01	0.11	0.1	0.13	0.13	0.14	0.12	0.06	0.2
2004/07/01	0.12	0.14	0.14	0.14	0.09	0.11	0.08	0.17
2005/01/01	0.17	0.11	0.2	0.09	0.07	0.14	0.05	0.17
2005/07/01	0.19	0.11	0.2	0.12	0.05	0.09	0.08	0.16
2006/01/01	0.12	0.2	0.21	0.1	0.06	0.06	0.07	0.17

Table A.1: Weights Computed for Highest-Ranked Strategy

Date	Factor							
	F1	F2	F3	F4	F5	F6	F7	F8
1998/01/01	0.21	0.12	0.22	0.14	0.06	0.11	0.08	0.05
1998/07/01	0.19	0.04	0.45	0.07	0.05	0.09	0.08	0.02
1999/01/01	0.25	0.05	0.3	0.16	0.1	0.09	0.02	0.03
1999/07/01	0.18	0.08	0.37	0.13	0.09	0.1	0.03	0.03
2000/01/01	0.22	0.06	0.36	0.13	0.11	0.06	0.02	0.04
2000/07/01	0.16	0.08	0.33	0.11	0.08	0.13	0.04	0.07
2001/01/01	0.12	0.09	0.37	0.09	0.12	0.09	0.04	0.09
2001/07/01	0.19	0.14	0.37	0.05	0.05	0.07	0.04	0.08
2002/01/01	0.14	0.08	0.61	0.03	0.04	0.04	0.03	0.03
2002/07/01	0.12	0.13	0.35	0.08	0.07	0.1	0.08	0.06
2003/01/01	0.12	0.18	0.12	0.1	0.12	0.19	0.06	0.11
2003/07/01	0.09	0.16	0.08	0.07	0.15	0.19	0.08	0.17
2004/01/01	0.1	0.14	0.08	0.08	0.21	0.21	0.1	0.08
2004/07/01	0.11	0.15	0.12	0.07	0.22	0.13	0.12	0.09
2005/01/01	0.1	0.2	0.1	0.1	0.17	0.16	0.06	0.12
2005/07/01	0.11	0.19	0.15	0.08	0.13	0.12	0.08	0.14
2006/01/01	0.11	0.14	0.24	0.12	0.06	0.08	0.13	0.13

Table A.2: Weights Computed for Lowest-Ranked Strategy

Appendix B
Simulations Monthly Returns

The following tables show the monthly returns of the highest- and lowest-ranked portfolio simulations.

	Portfolio		
Month	Highest-Ranked	Lowest-Ranked	MSCI REIT Index
1998/01	0.75%	0.46%	-1.24%
1998/02	-1.29%	-2.56%	-1.61%
1998/03	3.26%	1.30%	2.37%
1998/04	-3.45%	-3.15%	-3.54%
1998/05	0.77%	0.48%	-0.87%
1998/06	0.08%	-0.40%	-0.01%
1998/07	-3.65%	-6.73%	-7.02%
1998/08	-6.95%	-12.93%	-9.42%
1998/09	10.44%	6.46%	6.19%
1998/10	-1.17%	-0.38%	-1.89%
1998/11	-0.14%	3.87%	1.57%
1998/12	2.06%	-5.23%	-1.78%

Table B.1: Monthly Return for 1998

Month	Portfolio Highest-Ranked	Portfolio Lowest-Ranked	MSCI REIT Index
1999/01	-3.45%	-1.57%	-2.69%
1999/02	-4.05%	-1.54%	-1.64%
1999/03	1.30%	0.24%	-0.55%
1999/04	12.06%	9.21%	9.67%
1999/05	2.79%	1.60%	2.12%
1999/06	-0.51%	-2.95%	-1.86%
1999/07	-4.88%	-4.98%	-3.15%
1999/08	0.73%	-1.02%	-0.96%
1999/09	-2.73%	-5.25%	-4.18%
1999/10	0.51%	-1.33%	-2.28%
1999/11	-3.48%	-1.39%	-1.48%
1999/12	3.15%	3.98%	3.11%
2000/01	0.44%	-0.08%	0.62%
2000/02	-3.17%	-3.08%	-1.59%
2000/03	7.65%	5.67%	3.67%
2000/04	6.27%	5.25%	6.72%
2000/05	2.70%	0.46%	0.92%
2000/06	2.81%	4.46%	2.48%
2000/07	10.88%	5.33%	9.07%
2000/08	-2.57%	-0.37%	-4.10%
2000/09	4.23%	0.12%	3.08%
2000/10	-3.65%	-5.04%	-4.75%
2000/11	1.39%	3.55%	1.75%
2000/12	8.31%	3.53%	7.11%

Table B.2: Monthly Return for 1999 and 2000

	Portfolio		
Month	Highest-Ranked	Lowest-Ranked	MSCI REIT Index
2001/01	4.62%	-1.12%	0.43%
2001/02	-2.36%	-3.92%	-1.72%
2001/03	1.50%	-0.78%	0.82%
2001/04	4.58%	2.67%	2.32%
2001/05	4.78%	7.07%	2.24%
2001/06	2.82%	1.35%	6.03%
2001/07	0.33%	-5.29%	-2.16%
2001/08	1.78%	6.23%	3.70%
2001/09	-14.78%	-2.97%	-4%
2001/10	1.38%	-2.08%	-3.35%
2001/11	10.54%	7.34%	5.83%
2001/12	4.61%	1.66%	2.60%
2002/01	2.97%	-0.66%	-0.23%
2002/02	3.59%	2.16%	1.98%
2002/03	7.25%	5.50%	6.44%
2002/04	1.68%	-1.16%	0.64%
2002/05	0.18%	-0.45%	1.27%
2002/06	2.37%	1.51%	2.87%
2002/07	-4.95%	-5.35%	-5.61%
2002/08	-0.09%	-2.22%	0.17%
2002/09	-1.73%	-4.87%	-3.63%
2002/10	-6.64%	-1.89%	-5.02%
2002/11	5.12%	6.02%	4.61%
2002/12	0.99%	-0.67%	0.83%

Table B.3: Monthly Return for 2001 and 2002

Month	Portfolio Highest-Ranked	Portfolio Lowest-Ranked	MSCI REIT Index
2003/01	-2.06%	-7.46%	-2.75%
2003/02	3.96%	-7.90%	1.79%
2003/03	2.90%	3.20%	2.10%
2003/04	4.59%	6.15%	4.22%
2003/05	5.30%	12.80%	5.65%
2003/06	3.78%	0.42%	2.32%
2003/07	5.11%	11.29%	5.31%
2003/08	0.98%	0.61%	0.61%
2003/09	3.96%	6.09%	3.53%
2003/10	1.28%	0.08%	1.69%
2003/11	4.45%	1.54%	4.39%
2003/12	3.81%	5.13%	3.14%
2004/01	4.95%	5.55%	4.38%
2004/02	2.50%	-4.28%	1.67%
2004/03	5.97%	5.27%	5.58%
2004/04	-14.54%	-11.37%	-14.82%
2004/05	7.11%	5.95%	7.18%
2004/06	2.65%	6.14%	2.83%
2004/07	1.44%	-3.73%	0.53%
2004/08	6.87%	5.19%	8.03%
2004/09	0.02%	-0.52%	-0.20%
2004/10	4.67%	5.90%	5.47%
2004/11	5.70%	7.42%	4.24%
2004/12	5.11%	10.17%	4.89%

Table B.4: Monthly Return for 2003 and 2004

Month	Portfolio Highest-Ranked	Portfolio Lowest-Ranked	MSCI REIT Index
2005/01	-8.34%	-7.85%	-8.61%
2005/02	2.61%	-1.87%	2.92%
2005/03	-1.85%	-2.76%	-1.60%
2005/04	6.76%	1.40%	5.94%
2005/05	4.91%	8.53%	3.26%
2005/06	5.36%	3.85%	5.02%
2005/07	6.95%	5.47%	7.17%
2005/08	-3.03%	-2.46%	-3.85%
2005/09	0.93%	-0.27%	0.57%
2005/10	-4.23%	-1.90%	-2.38%
2005/11	6.57%	6.05%	4.33%
2005/12	-0.15%	1.96%	-0.10%
2006/01	5.85%	9.22%	7.68%
2006/02	3.46%	1.20%	1.88%
2006/03	6.81%	4.51%	4.97%
2006/04	-2.90%	-2.49%	-3.72%
2006/05	-1.82%	-0.83%	-2.88%
2006/06	5.01%	3.56%	5.38%

Table B.5: Monthly Return for 2005 and 2006

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