Data Structures and Algorithms

COMP 251, Winter 2013

Assignment 5

Due date: Tuesday, April 16, 2013
8pm

Exams for all courses are held at the end of each term, within a fixed number of days, and using a fixed number of rooms (assume each room has capacity to hold any one course). Each day has 3 time slots available for exams (morning, afternoon, evening). The task is then to compute an exam schedule given as input a set of courses C, a set of exam days D, and rooms R, and producing as output a time slot and room for each exam.

In all of the below, in expressing pseudo-code you may assume you already have a function `FordFulkerson(`G`)` which computes maximum flow on a weighted, directed graph given as an argument.

Let $n = |C|$, $m = |D|$, and $r = |R|$.

1. Describe how to find a valid exam schedule as a maximum flow problem. As well as a clearly labelled, visual representation of the flow network (drawn as for some small n, m, r, but clear how that would extend to arbitrary n, m, r), provide pseudo-code which would properly construct the flow network, call `FordFulkerson()`, and then emit the schedule. **5**

2. A realistic exam schedule must take into account exam conflicts. If any student is taking course c_1 and course c_2, then the exams for c_1 and c_2 cannot be held at the same time. **10**

   Modify your solution to accommodate this constraint, assuming that your input now includes a set of students S and a mapping from students to course registrations ($S \rightarrow \mathcal{P}(C)$). Again, provide a representative flow network where it is clear how it extends to arbitrary input, as well as the full pseudo-code which constructs the appropriate flow network, runs it through `FordFulkerson()`, and emits a valid schedule.

3. The goal now is to understand the complexity of your solution to question 2.

   (a) How many edges are in the network you constructed in question 2? Give a precise number (and justification), based on n, m, r, and any other parametrization(s) of the $S \rightarrow \mathcal{P}(C)$ function you feel is necessary. **10**

   (b) What are upper and lower asymptotic bounds on the time complexity of your code, excluding the actual maximum flow calculation? Give a clear and convincing derivation. **5**

4. Ideally, the exam schedule should also be as short as possible. Use pseudo-code to describe an algorithm that would find the minimum m given all of the above constraints. Suppose you also want it to be $O(n^k)$ for some constant k. Does this limit the other parameters—what complexity classes would m, r and the set of student conflicts have to be in in order to ensure this is possible? Clearly justify your answer. **10**

What to hand in

Submit your assignment to *MyCourses*. Note that clock accuracy varies, and late assignments will not be accepted without a medical note: **do not wait until the last minute**. Assignments must be submitted on the due date **before 8pm**.

Submit either an ASCII text document or a .pdf file *with all fonts embedded*. Do not submit .doc or .docx files. Images (plots or scans) are acceptable in all common graphic file formats.

Note that for written answers you must show all intermediate work to receive full marks.
This assignment is worth 6% of your final grade. **40**