A structure-driven performance analysis of sparse matrix-vector multiplication

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Outline

1. Introduction
2. Experimental Design
3. Research Questions : Effect of Matrix Structure
   - On the Choice of Storage Format
   - Within a Storage Format
   - Along with Hardware Characteristics
4. Summary and Future Work

Outline

Background : Sparse Matrix Storage Formats

- **A sparse matrix :** a matrix in which most of the elements are zero.
- **Basic sparse storage formats :**
  - Coordinate Format (COO)
  - Compressed Sparse Row Format (CSR)
  - Diagonal Format (DIA)
  - ELLPACK Format (ELL)

A

1	0	6	0
0	2	0	7
0	0	3	0
5	0	0	4

COO :

row	0	0	1	1	2	3	3
col	0	2	1	3	2	0	3
val	1	6	2	7	3	5	4

CSR :

row_ptr	0	2	4	5	7		
col	0	2	1	3	2	0	3
val	1	6	2	7	3	5	4

DIA :

data

-	-	-	5
1	2	3	4
6	7	-	-

offset

-3	0	2

ELL :

data

1	2	3	5
6	7	-	4

indices

0	1	2	0
2	3	-	3

Background : SpMV

Sparse Matrix-Vector Multiplication

- $y = Ax$, where A is a sparse matrix and the input vector x and output vector y are dense.
- Working set size : sizeof(A) + sizeof(x) + sizeof(y)

$$\underset{A}{\begin{bmatrix} 1 & 0 & 6 & 0 \\ 0 & 2 & 0 & 7 \\ 0 & 0 & 3 & 0 \\ 5 & 0 & 0 & 4 \end{bmatrix}} * \underset{x}{\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}} = \underset{y}{\begin{bmatrix} 7 \\ 9 \\ 3 \\ 9 \end{bmatrix}}$$

Why Sparse Matrices on the Web?

- Web-enabled devices everywhere!
- Various compute-intensive applications involving sparse matrices on the web.
  - Image editing
  - Computer-aided design
  - Text classification (data mining)
  - Deep learning
- Recent addition of WebAssembly to the world of JavaScript.

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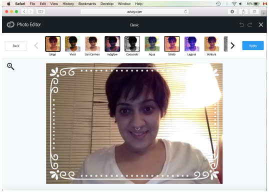

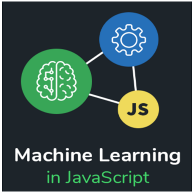

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Why SpMV is so Important?

- A computational kernel used in many scientific and machine learning applications.
- occurs frequently in these applications.
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How to Optimize SpMV Performance

1. Select an optimal format to store the input sparse matrix.
2. Apply data and low-level code optimizations to a single format.

Depends on the structure of the matrix and the machine characteristics.

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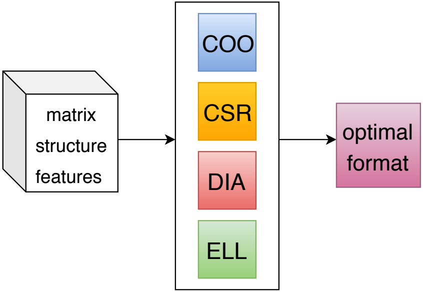
Our Goal

To understand the effect of :

1. matrix structure on the choice of storage format.
2. matrix structure on the SpMV performance within a storage format.
3. interaction between matrix structure and hardware characteristics on the SpMV performance.



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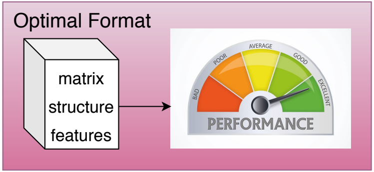

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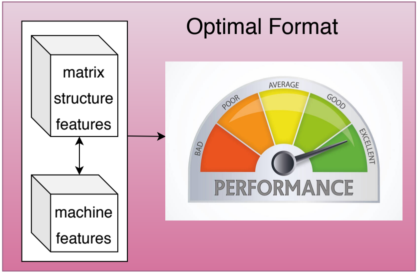


Outline

Reference Implementations and Measurement Setup

Developed a reference set of sequential C and hand-tuned WebAssembly implementations of SpMV for different formats on same algorithmic lines.

```
void spmv_coo(int *row, int *col, float *val, int nnz, int N, float *x, float *y)
{ int i;
  for(i = 0; i < nnz ; i++)
    y[row[i]] += val[i] * x[col[i]];
}
```

Listing 1: Single-precision SpMV COO implementation in C

- **Benchmarks** : Around 2000 real-life sparse matrices from The SuiteSparse Matrix Collection.
- **Sparse Storage Formats** : COO, CSR, DIA, ELL
- Measured SpMV Performance for C and WebAssembly in FLOPS (Floating point operations per second).

Target Languages and Runtime

- **Machine Architecture**

Intel Core i7-3930K with 6 3.20GHz cores, 12MB last-level cache and 16GB memory,running Ubuntu Linux 16.04.2

- **C**

Compiled with gcc version 7.2.0 at optimization level -O3

- **WebAssembly**

Used Chrome 74 browser (Official build 74.0.3729.108 with V8 JavaScript engine 7.4.288.25) as the execution environment with –experimental-wasm-simd flag to enable the use of SIMD instructions.

How we chose the optimal format?

x%-affinity

We say that an input matrix A has an x%-affinity for storage format F, if the performance for F is at least x% better than all other formats and the performance difference is greater than the measurement error.

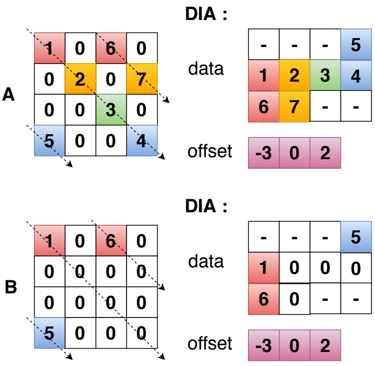
Example

For example, if input array A in format CSR, is more than 10% faster than input A in all other formats, and 10% is more than the measurement error, then we say that A has a 10%-affinity for CSR.

Outline

Matrix Structure Feature : *dia_ratio*

- $dia\_ratio = \dfrac{ndiag\_elems}{nnz}$ where, **nnz** : number of non-zeros, **ndiag_elems** : number of elements in the diagonals
- Indicates if the given matrix is a good fit for DIA format or not.
- dia_ratio(A) = 7/7 = 1
- dia_ratio(B) = 7/3 = 2.33



DIA Format

Matrices with dia_ratio <= 3 show affinity towards the DIA format, except for a few matrices.

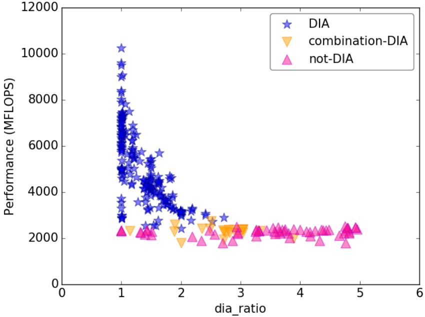


Figure: C

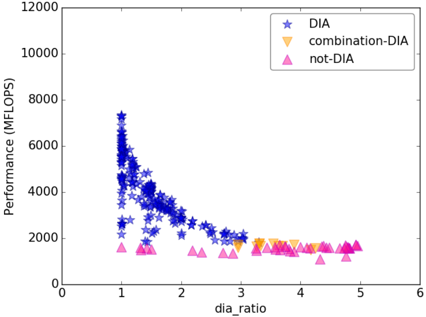


Figure: Wasm

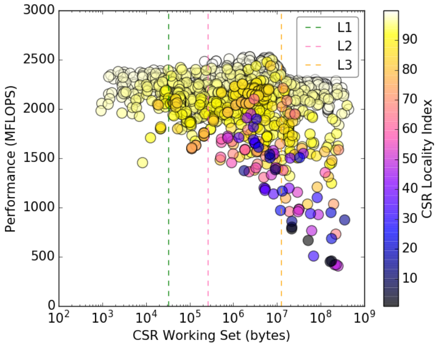
Relationship between Storage Format and Structure Features

Format	Feature(s)	Priority
DIA	*dia_ratio* $\leq$ 3 and large N	1
ELL	*ell_ratio* $\simeq$ 1 and small *max_nnz_per_row*	2
COO	nnz $<$ N or small *avg_nnz_per_row* and uneven number of non-zeros per row	3

Outline

SpMV Performance within CSR Matrices

- CSR Working Set : (N+1) + 2*nnz + 2*N
- Irregular access for vector x affects performance.
- Introduced some new matrix structure features : *ELL Locality Index, CSR Locality Index*
- Based on data locality model
- Using reuse-distance concept



CSR Locality Index

indicator of irregular memory access for vector x for a CSR matrix.

CSR Locality Index : Step 1

- Calculate Row Reuse Distance for each non-zero.
- Row Reuse Distance (rrd) : Distance from the last non-zero whose column index corresponds to the same cache line of the input vector x.
- Unit of distance : rows

*Assume the cache line size to be 2 and cache size to be fixed for this example.

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x

1	cache line 1
1	
1	cache line 2
1	

CSR :

row_ptr	0	2	4	5	7		
col	0	2	1	3	2	0	3
val	1	6	2	7	3	5	4
x-vector access pattern	x[0]	x[2]	x[1]	x[3]	x[2]	x[0]	x[3]
rrd	-	-	1	1	1	2	1

CSR Locality Index : Step 2

- Calculate CSR Reuse Distance using frequency distribution over Row Reuse Distance (rrd).
- CSR Reuse Distance[p] : the number of non-zeros of sparse matrix A stored in the CSR format which access the input vector x with p Row Reuse Distance.

CSR :

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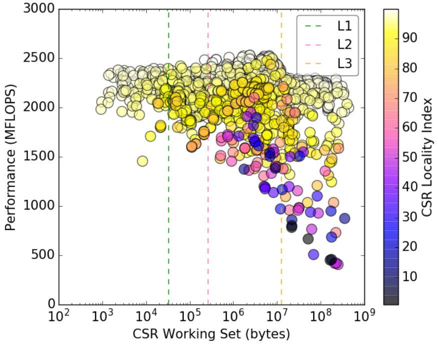
col	0	2	1	3	2	0	3
val	1	6	2	7	3	5	4
x-vector access pattern	x[0]	x[2]	x[1]	x[3]	x[2]	x[0]	x[3]
rrd	-	-	1	1	1	2	1

Index	0	1	2	3
Frequency	0	4	1	0

CSR Locality Index : Step 3

- Calculate CSR Locality Index using cumulative percentage over CSR Reuse Distance.
- $\mathit{CSR\ Locality\ Index} = \dfrac{\sum_{p=0}^{15} \mathit{CSR\ Reuse\ Distance}[p]}{nnz} \times 100$
- This feature accounts for :
  - spatial locality for the non-zeros in a row.
  - temporal locality for the non-zeros in the neighbouring rows.

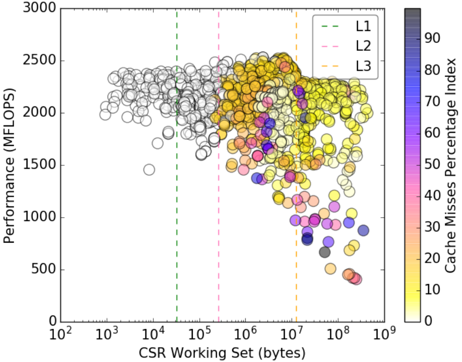
* We chose the limit to be 15 based on our experiments



Outline

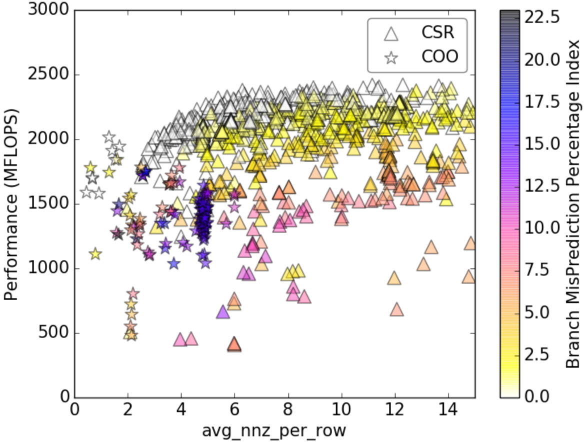
Cache Memory : CSR Performance

- Features based on data locality model have their roots in the hardware features like data cache misses.
- Measured true performance counters using PAPI tool.
- $Index = \frac{PAPI\_L1\_DCM \vee PAPI\_L2\_DCM \vee PAPI\_L3\_TCM}{nnz} \times 100$



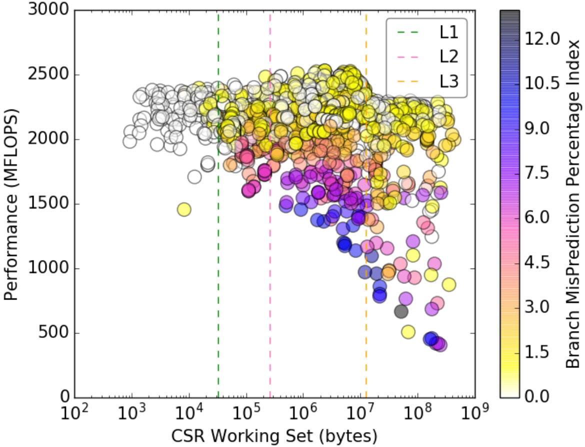
Branch Prediction Unit : CSR vs COO

- $Index = \frac{PAPI\_BR\_MSP}{PAPI\_BR\_PRC + PAPI\_BR\_MSP} \times 100$



Branch Prediction Unit : CSR Performance

- $Index = \frac{PAPI\_BR\_MSP}{PAPI\_BR\_PRC + PAPI\_BR\_MSP} \times 100$



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Summary

- The optimal choice of storage format is governed both by the structure of the matrix and the code optimization opportunities available.
- Due to different code generation strategy, the SpMV performance suffers in the case of WebAssembly for Chrome (v8) browser.
- Our data locality based structure features estimate if the SpMV performance is affected by the irregular memory accesses for vector x.
- We validate our evaluations and parameter choices using hardware performance counters.

Future Work

- Further explore to quantify the impact of additional hardware features on SpMV performance via matrix structure features.
- Explore new optimization opportunities for hand-tuned WebAssembly implementations through the upcoming WebAssembly instructions.
- Develop parallel versions of SpMV based on multithreading features like web workers.
- Develop automatic techniques to choose the best format for web-based SpMV.

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