A structure-driven performance analysis of sparse matrix-vector multiplication

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Outline

1 Introduction

2 Experimental Design

3 Research Questions: Effect of Matrix Structure
   - On the Choice of Storage Format
   - Within a Storage Format
   - Along with Hardware Characteristics

4 Summary and Future Work
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4 Summary and Future Work
A sparse matrix: a matrix in which most of the elements are zero.

Basic sparse storage formats:
- Coordinate Format (COO)
- Compressed Sparse Row Format (CSR)
- Diagonal Format (DIA)
- ELLPACK Format (ELL)

\[
A = \begin{bmatrix}
1 & 0 & 6 & 0 \\
0 & 2 & 0 & 7 \\
0 & 0 & 3 & 0 \\
5 & 0 & 0 & 4
\end{bmatrix}
\]

**COO:**
- row: 0 0 1 1 2 3 3
- col: 0 2 1 3 2 0 3
- val: 1 6 2 7 3 5 4

**CSR:**
- row_ptr: 0 2 4 5 7
- col: 0 2 1 3 2 0 3
- val: 1 6 2 7 3 5 4

**DIA:**
- data: 5
- offset: -3 0 2

**ELL:**
- data: 1 2 3 4 6 7 
- indices: 0 1 2 0 2 3 - 3
Sparse Matrix-Vector Multiplication

- $y = Ax$, where $A$ is a sparse matrix and the input vector $x$ and output vector $y$ are dense.
- Working set size: $\text{sizeof}(A) + \text{sizeof}(x) + \text{sizeof}(y)$
Why Sparse Matrices on the Web?

- Web-enabled devices everywhere!
- Various compute-intensive applications involving sparse matrices on the web.
  - Image editing
  - Computer-aided design
  - Text classification (data mining)
  - Deep learning
- Recent addition of WebAssembly to the world of JavaScript.
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Why SpMV is so Important?

- A computational kernel used in many scientific and machine learning applications.
- Occurs frequently in these applications.
- Hence, a good candidate for their performance optimization.
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How to Optimize SpMV Performance

1. Select an optimal format to store the input sparse matrix.

2. Apply data and low-level code optimizations to a single format.

Depends on the structure of the matrix and the machine characteristics.
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Depends on the structure of the matrix and the machine characteristics.
Our Goal

To understand the effect of:

1. matrix structure on the choice of storage format.
2. matrix structure on the SpMV performance within a storage format.
3. interaction between matrix structure and hardware characteristics on the SpMV performance.
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Developed a reference set of sequential C and hand-tuned WebAssembly implementations of SpMV for different formats on same algorithmic lines.

```c
void spmv_coo (int *row , int *col , float *val , int nnz , int N, float *x, float *y)
{ int i;
  for (i = 0; i < nnz ; i++)
    y[row[i]] += val[i] * x[col[i]];
}
```

**Listing 1:** Single-precision SpMV COO implementation in C

- **Benchmarks**: Around 2000 real-life sparse matrices from The SuiteSparse Matrix Collection.
- **Sparse Storage Formats**: COO, CSR, DIA, ELL
- Measured SpMV Performance for C and WebAssembly in FLOPS (Floating point operations per second).
Target Languages and Runtime

- **Machine Architecture**
  
  Intel Core i7-3930K with 6 3.20GHz cores, 12MB last-level cache and 16GB memory, running Ubuntu Linux 16.04.2

- **C**
  
  Compiled with gcc version 7.2.0 at optimization level -O3

- **WebAssembly**
  
  Used Chrome 74 browser (Official build 74.0.3729.108 with V8 JavaScript engine 7.4.288.25) as the execution environment with –experimental-wasm-simd flag to enable the use of SIMD instructions.
How we chose the optimal format?

x%-affinity

We say that an input matrix A has an x%-affinity for storage format F, if the performance for F is at least x% better than all other formats and the performance difference is greater than the measurement error.

Example

For example, if input array A in format CSR, is more than 10% faster than input A in all other formats, and 10% is more than the measurement error, then we say that A has a 10%-affinity for CSR.
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Matrix Structure Feature: \textit{dia\_ratio}

- \textit{dia\_ratio} = \frac{\text{ndiag\_elems}}{\text{nnz}}

where, \texttt{nnz}: number of non-zeros, \texttt{ndiag\_elems}: number of elements in the diagonals

- Indicating if the given matrix is a good fit for DIA format or not.

- \texttt{dia\_ratio(A)} = \frac{7}{7} = 1

- \texttt{dia\_ratio(B)} = \frac{7}{3} = 2.33
Matrices with $\text{dia\_ratio} \leq 3$ show affinity towards the DIA format, except for a few matrices.

**Figure: C**

**Figure: Wasm**
### Relationship between Storage Format and Structure Features

<table>
<thead>
<tr>
<th>Format</th>
<th>Feature(s)</th>
<th>Priority</th>
</tr>
</thead>
<tbody>
<tr>
<td>DIA</td>
<td>$dia_ratio \leq 3$ and large $N$</td>
<td>1</td>
</tr>
<tr>
<td>ELL</td>
<td>$ell_ratio \simeq 1$ and small $max_nnz_per_row$</td>
<td>2</td>
</tr>
<tr>
<td>COO</td>
<td>$nnz &lt; N$ or small $avg_nnz_per_row$ and uneven number of non-zeros per row</td>
<td>3</td>
</tr>
</tbody>
</table>
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SpMV Performance within CSR Matrices

- CSR Working Set: \((N+1) + 2*\text{nnz} + 2*N\)
- Irregular access for vector \(x\) affects performance.
- Introduced some new matrix structure features: ELL, Locality Index, CSR Locality Index
- Based on data locality model
- Using reuse-distance concept

CSR Locality Index

indicator of irregular memory access for vector \(x\) for a CSR matrix.
Calculate Row Reuse Distance for each non-zero.

Row Reuse Distance (rrd): Distance from the last non-zero whose column index corresponds to the same cache line of the input vector x.

Unit of distance: rows

*Assume the cache line size to be 2 and cache size to be fixed for this example.
Calculate Row Reuse Distance for each non-zero.

Row Reuse Distance (rrd) : Distance from the last non-zero whose column index corresponds to the same cache line of the input vector x.

Unit of distance : rows

*Assume the cache line size to be 2 and cache size to be fixed for this example.
Calculate CSR Reuse Distance using frequency distribution over Row Reuse Distance (rrd).

CSR Reuse Distance \([p]\) : the number of non-zeros of sparse matrix \(A\) stored in the CSR format which access the input vector \(x\) with \(p\) Row Reuse Distance.

### CSR:

<table>
<thead>
<tr>
<th>row_ptr</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>5</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>col</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>val</td>
<td>1</td>
<td>6</td>
<td>2</td>
<td>7</td>
<td>3</td>
</tr>
</tbody>
</table>

#### x-vector access pattern

\(x[0], x[2], x[1], x[3], x[2], x[0], x[3]\)

#### rrd

- - 1 1 1 2 1

<table>
<thead>
<tr>
<th>Index</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>0</td>
<td>4</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>
Calculate CSR Locality Index using cumulative percentage over CSR Reuse Distance.

\[
\text{CSR Locality Index} = \frac{\sum_{p=0}^{15} \text{CSR Reuse Distance}[p]}{\text{nnz}} \times 100
\]

This feature accounts for:
- spatial locality for the non-zeros in a row.
- temporal locality for the non-zeros in the neighbouring rows.

* We chose the limit to be 15 based on our experiments.
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Features based on data locality model have their roots in the hardware features like data cache misses.

Measured true performance counters using PAPI tool.

\[ \text{Index} = \frac{\text{PAPI}_{L1\_DCM} \lor \text{PAPI}_{L2\_DCM} \lor \text{PAPI}_{L3\_TCM}}{\text{nnz}} \times 100 \]
Branch Prediction Unit: CSR vs COO

\[ \text{Index} = \frac{\text{PAPI}_{\text{BR-MSP}}}{\text{PAPI}_{\text{BR-PRC}} + \text{PAPI}_{\text{BR-MSP}}} \times 100 \]
\[ \text{Index} = \frac{PAPI_{BR\_MSP}}{PAPI_{BR\_PRC} + PAPI_{BR\_MSP}} \times 100 \]
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Summary and Future Work
The optimal choice of storage format is governed both by the structure of the matrix and the code optimization opportunities available.

Due to different code generation strategy, the SpMV performance suffers in the case of WebAssembly for Chrome (v8) browser.

Our data locality based structure features estimate if the SpMV performance is affected by the irregular memory accesses for vector $x$.

We validate our evaluations and parameter choices using hardware performance counters.
Future Work

- Further explore to quantify the impact of additional hardware features on SpMV performance via matrix structure features.
- Explore new optimization opportunities for hand-tuned WebAssembly implementations through the upcoming WebAssembly instructions.
- Develop parallel versions of SpMV based on multithreading features like web workers.
- Develop automatic techniques to choose the best format for web-based SpMV.

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