Control Flow Analysis

COMP 621 – Program Analysis and Transformations

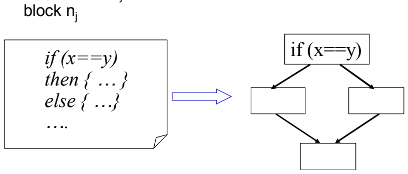
These slides have been adapted from
http://cs.gmu.edu/~white/CS640/Slides/CS640-2-02.ppt
by Professor Liz White.

Program Control Flow

- Control flow
  - Sequence of operations
  - Representations
    - Control flow graph
    - Control dependence
    - Call graph
- Control flow analysis
  - Analyzing program to discover its control structure
  - Today's topic: CFG-based analysis

Control Flow Graph

- CFG models flow of control in the program (procedure)
- G = (N, E) as a directed graph
  - Node n ∈ N: basic blocks
    - A basic block is a maximal sequence of stmts with a single entry point, single exit point, and no internal branches
    - For simplicity, we assume a unique entry node n_0 and a unique exit node n_f in later discussions
  - Edge e=(n_i, n_j) ∈ E: possible transfer of control from block n_i to block n_j



Basic Blocks

- Definition
  - A basic block is a maximal sequence of consecutive statements with a single entry point, a single exit point, and no internal branches
- Basic unit in control flow analysis
- Local level of code optimizations
  - Redundancy elimination
  - Register-allocation

Basic Block Example

```
(1)   i := m – 1
(2)   j := n
(3)   t1 := 4 * n
(4)   v := a[t1]
(5)   i := i + 1
(6)   t2 := 4 * i
(7)   t3 := a[t2]
(8)   if t3 < v goto (5)
(9)   j := j – 1
(10)  t4 := 4 * j
(11)  t5 := a[t4]
(12)  if t5 > v goto (9)
(13)  if i >= j goto (23)
(14)  t6 := 4*i
(15)  x := a[t6]
…
```

- How many basic blocks in this code fragment?
- What are they?

Basic Block Example

```
(1)   i := m – 1
(2)   j := n
(3)   t1 := 4 * n
(4)   v := a[t1]
(5)   i := i + 1
(6)   t2 := 4 * I
(7)   t3 := a[t2]
(8)   if t3 < v goto (5)
(9)   j := j – 1
(10)  t4 := 4 * j
(11)  t5 := a[t4]
(12)  if t5 > v goto (9)
(13)  if i >= j goto (23)
(14)  t6 := 4*I
(15)  x := a[t6]
…
```

- How many basic blocks in this code fragment?
- What are they?

Identify Basic Blocks

Input: A sequence of intermediate code statements

1. Determine the *leaders*, the first statements of basic blocks
   - The first statement in the sequence (entry point) is a leader
   - Any statement that is the target of a branch (conditional or unconditional) is a leader
   - Any statement immediately following a branch (conditional or unconditional) or a return is a leader
2. For each leader, its basic block is the leader and all statements up to, but not including, the next leader or the end of the program

Example

(1) i := m – 1
(2) j := n
(3) t1 := 4 * n
(4) v := a[t1]
(5) i := i + 1
(6) t2 := 4 * i
(7) t3 := a[t2]
(8) if t3 < v goto (5)
(9) j := j - 1
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(12) If t5 > v goto (9)
(13) if i >= j goto (23)
(14) t6 := 4*i
(15) x := a[t6]
(16) t7 := 4 * i
(17) t8 := 4 * j
(18) t9 := a[t8]
(19) a[t7] := t9
(20) t10 := 4 * j
(21) a[t10] := x
(22) goto (5)
(23) t11 := 4 * i
(24) x := a[t11]
(25) t12 := 4 * i
(26) t13 := 4 * n
(27) t14 := a[t13]
(28) a[t12] := t14
(29) t15 := 4 * n
(30) a[t15] := x

Example: Leaders

(1) i := m – 1
(2) j := n
(3) t1 := 4 * n
(4) v := a[t1]
(5) i := i + 1
(6) t2 := 4 * i
(7) t3 := a[t2]
(8) if t3 < v goto (5)
(9) j := j - 1
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(15) x := a[t6]
(16) t7 := 4 * i
(17) t8 := 4 * j
(18) t9 := a[t8]
(19) a[t7] := t9
(20) t10 := 4 * j
(21) a[t10] := x
(22) goto (5)
(23) t11 := 4 * i
(24) x := a[t11]
(25) t12 := 4 * i
(26) t13 := 4 * n
(27) t14 := a[t13]
(28) a[t12] := t14
(29) t15 := 4 * n
(30) a[t15] := x

Example: Basic Blocks

(1) i := m – 1
(2) j := n
(3) t1 := 4 * n
(4) v := a[t1]

(5) i := i + 1
(6) t2 := 4 * i
(7) t3 := a[t2]
(8) if t3 < v goto (5)

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(10) t4 := 4 * j
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(14) t6 := 4*i
(15) x := a[t6]
(16) t7 := 4 * i
(17) t8 := 4 * j
(18) t9 := a[t8]
(19) a[t7] := t9
(20) t10 := 4 * j
(21) a[t10] := x
(22) goto (5)

(23) t11 := 4 * i
(24) x := a[t11]
(25) t12 := 4 * i
(26) t13 := 4 * n
(27) t14 := a[t13]
(28) a[t12] := t14
(29) t15 := 4 * n
(30) a[t15] := x

Generating CFGs

- Partition intermediate code into basic blocks
- Add edges corresponding to control flows between blocks
  - Unconditional goto
  - Conditional branch – multiple edges
  - Sequential flow – control passes to the next block (if no branch at the end)
- If no unique entry node n_0 or exit node n_f, add dummy nodes and insert necessary edges
  - Ideally no edges entering n_0; no edges exiting n_f
  - Simplify many analysis and transformation algorithms

Example: CFG

(1) i := m – 1
(2) j := n
(3) t1 := 4 * n
(4) v := a[t1]

(5) i := i + 1
(6) t2 := 4 * i
(7) t3 := a[t2]
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(21) a[t10] := x
(22) goto (5)

(23) t11 := 4 * i
(24) x := a[t11]
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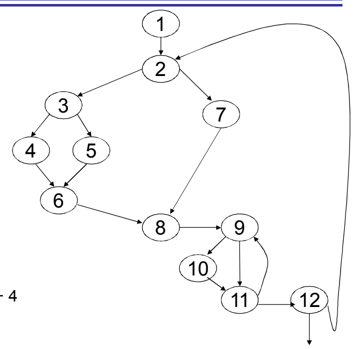
(27) t14 := a[t13]
(28) a[t12] := t14
(29) t15 := 4 * n
(30) a[t15] := x

CFG and HL code

```
I = 1
J = 1
K = 1
L = 1
repeat
  if (P) then begin
     J = I
     if (Q) then L = 2
     else L = 3
     K = K + 1
  end
  else K = K + 2
  print (I,J,K,L)
  repeat
     if (R) then L = L + 4
  until (S)
  I = I + 6
until (T)
```

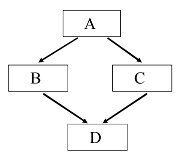


Complications in CFG Construction

- Function calls
  - Instruction scheduling may prefer function calls as basic block boundaries
  - Special functions as setjmp() and longjmp()
- Exception handling
- Ambiguous jump
  - Jump r1 //target stored in register r1
  - Static analysis may generate edges that never occur at runtime
    - Record potential targets if possible
- Jumps target outside the current procedure
  - PASCAL, Algol: still restricted to lexically enclosing procedure

Nodes in CFG

- Given a CFG = <N, E>
  - If there is an edge $n_i \rightarrow n_j \in E$
    - n_i is a predecessor of n_j
    - n_j is a successor of n_i
  - For any node $n \in N$
    - Pred(n): the set of predecessors of n
    - Succ(n): the set of successors of n
    - A branch node is a node that has more than one successor
    - A join node is a node that has more than one predecessor



Depth First Traversal

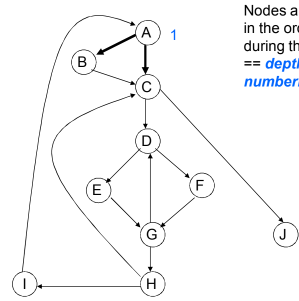
- CFG is a rooted, directed graph
  - Entry node as the root
- Depth-first traversal (depth-first searching)
  - Idea: start at the root and explore as far/deep as possible along each branch before backtracking
  - Can build a spanning tree for the graph
- Spanning tree of a directed graph G contains all nodes of G such that
  - There is a path from the root to any node reachable in the original graph and
  - There are no cycles

DFS Spanning Tree Algorithm

```
procedure span(v)  /* v is a node in the
  graph */
   InTree(v) = true
   For each w that is a successor of v do
        if (!InTree(w)) then
        Add edge v → w to spanning tree
        span(w)
end span
```

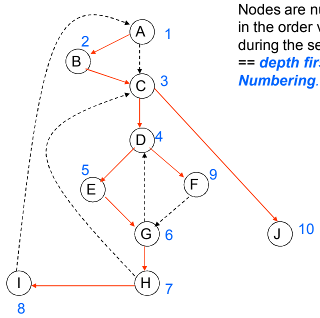
- Initial: $\texttt{span}(n_0)$

DFST Example



Nodes are numbered in the order visited during the search == ***depth first pre-order numbering***.

DFST Example



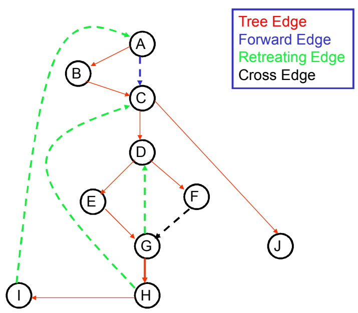
Nodes are numbered in the order visited during the search == ***depth first pre-order Numbering***.

CFG Edges Classification

Edge $x \rightarrow y$ in a CFG is an

- *Advancing edge* – if x is an ancestor of y in the tree
  - *Tree edge* – if part of the spanning tree
  - *Forward edge* – if not part of the spanning tree and x is an ancestor of y in the tree
- Retreating edge – if not part of the spanning tree and y is an ancestor of x in the tree
- *Cross edge* – if not part of the spanning tree and neither is an ancestor of the other

DFST Example

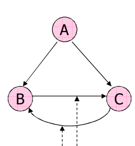


Back Edges and Reducibility

- An edge $x \rightarrow y$ in a CFG is a *back edge* if every path from the entry node of the flow graph to x goes through y
  - y dominates x : more details later
  - Every back edge is a retreating edge
  - Vice versa?
- A flow graph is *reducible* if all its retreating edges in any DFST are also back edges
  - Flow graphs that occur in practice are almost always reducible

Non-Reducible Graphs

- Testing reducibility: Take any DFST for the flow graph, remove the back edges, and check that the result is acyclic



In any DFST, one of these edges will be a retreating edge

Nodes Ordering wrt DFST

- Enhanced depth-first spanning tree algorithm:

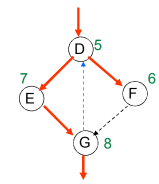
```
time =0;
procedure span(v)  /* v is a node in the graph */
    InTree(v) = true; d[v] = ++time;
    For each w that is a successor of v do
        if (!InTree(w)) then
            Add edge v → w to spanning tree
            span(w)
    f[v]=++time;
end span
```

- Associate two numbers to each node v in the graph
  - d[v]: discovery time of v in the spanning
  - f[v]: finish time of v in the spanning

Nodes Ordering wrt DFST

- Pre-ordering
  - Ordering of vertices based on discovery time
- Post-ordering
  - Ordering of vertices based on finish time
- Reverse post-ordering
  - The reverse of a post-ordering, i.e. ordering of vertices in the opposite order of their finish time
  - Not the same as pre-ordering
  - Commonly used in forward data flow analysis
    - Backward data flow analysis: RPO on the reverse CFG

Ordering Example



- Pre-ordering: DEGF
- Post-ordering: GEFD
- Reverse post-ordering: DFEG

Big Picture

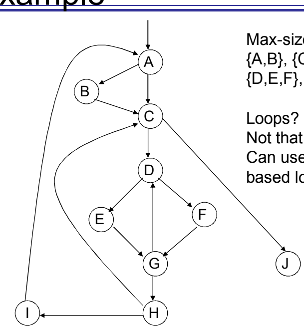
Why care about ordering / back edges?

- CFGs are commonly used to propagate information between nodes (basic blocks)
  - Data flow analysis
- The existence of back edges / cycles in flow graphs indicates that we may need to traverse the graph more than once
  - Iterative algorithms: when to stop? How quickly can we stop?
- Proper ordering of nodes during iterative algorithm assures number of passes limited by the number of "nested" back edges

Regions in CFG

- Extended basic block (EBB)
  - EBB is a maximal set of nodes in a CFG that contains no join nodes other than the entry node
    - A single entry and possibly multiple exits
    - Some optimizations like value numbering and instruction scheduling are more effective if applied in EBBs
- Natural loop
  - Loop is a collection of nodes in a CFG such that
    - All nodes in the collection are strongly connected, and
    - The collection of nodes has a unique *entry*: the only way to visit the loop from outside
  - A loop that contains no other loops is an *inner loop*
  - Main target of program optimizations

EBB Example



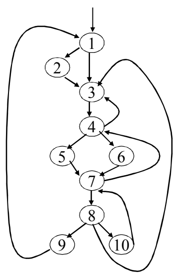
Max-size EBBs:
{A,B}, {C,J},
{D,E,F}, {G,H,I}

Loops?
Not that obvious…
Can use dominator-based loop detection

Dominance

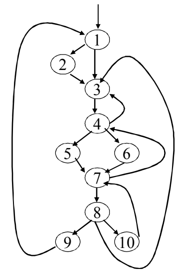
- Node *d* of a CFG *dominates* node *n* if every path from the entry node of the graph to *n* passes through *d* (*d dom n*)
  - Dom(*n*): the set of dominators of node *n*
  - Every node dominates itself: $n \in$ Dom(n)
  - Node *d* strictly dominates *n* if $d \in$ Dom(n) and $d \neq n$
  - Dominance-based loop recognition: entry of a loop dominates all nodes in the loop
- Each node *n* has a unique *immediate dominator* *m* which is the last dominator of *n* on any path from the entry to *n* (*m idom n*), $m \neq n$
  - The immediate dominator *m* of *n* is the strict dominator of *n* that is closest to *n*

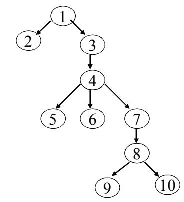
Dominator Example



Block	Dom	IDom
1	{1}	—
2	{1,2}	1
3	{1,3}	1
4	{1,3,4}	3
5	{1,3,4,5}	4
6	{1,3,4,6}	4
7	{1,3,4,7}	4
8	{1,3,4,7,8}	7
9	{1,3,4,7,8,9}	8
10	{1,3,4,7,8,10}	8

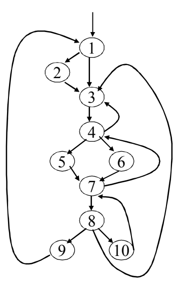
Dominator Trees





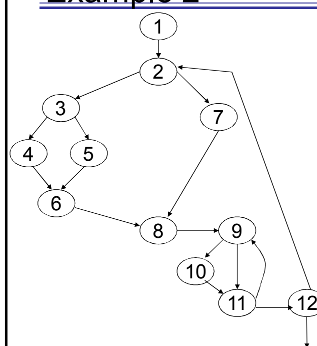
- In a dominator tree, a node's parent is its immediate dominator

Other sets of interest



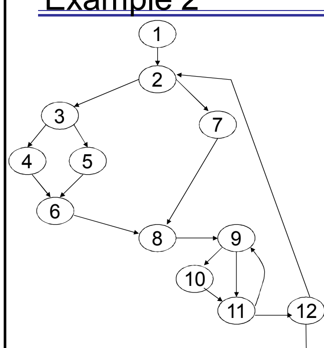
Block	SDom Dom-n	Dom^{-1}
1	{}	{1,2,3,4,5,6,7,8,9,10}
2	{1}	{2}
3	{1}	{3,4,5,6,7,8,9,10}
4	{1,3}	{4,5,6,7,8,9,10}
5	{1,3,4}	{5}
6	{1,3,4}	{6}
7	{1,3,4}	{7,8,9,10}
8	{1,3,4,7}	{8,9,10}
9	{1,3,4,7,8}	{9}
10	{1,3,4,7,8}	{10}

Example 2



Block	Dom	IDom
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		

Example 2



Block	Dom	IDom
1	1	-
2	1,2	1
3	1,2,3	2
4	1,2,3,4	3
5	1,2,3,5	3
6	1,2,3,6	3
7	1,2,7	2
8	1,2,8	2
9	1,2,8,9	8
10	1,2,8,9,10	9
11	1,2,8,9,11	9
12	1,2,8,9,11,12	11

Algorithm: Computing DOM

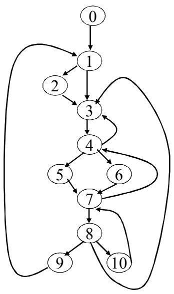
- An iterative fixed-point calculation

N is the set of nodes in the CFG
DOM(n_0) = {n_0} (n_0 is the entry)
For all nodes $x \neq n_0$
    DOM(x) = N

Until no more changes to dominator sets
    for all nodes $x \neq n_0$
        DOM(x) = { x } + ($\cap$ DOM(P)) for all predecessors P of x

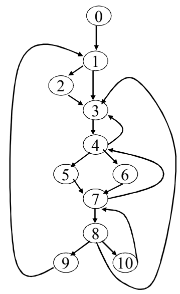
- At termination, node d in DOM(n) iff d dominates n

Dominator Example



	initial	iteration1
0	{0}	{0}
1	N	{1} + (Dom(0) ∩ Dom(9)) = {0,1}
2	N	{2} + Dom(1) = {0,1,2}
3	N	{3} + (Dom(1) ∩ Dom(2) ∩ Dom(8) ∩ Dom(4)) = {0,1,3}
4	N	{4} + (Dom(3) ∩ Dom(7)) = {0,1,3,4}
5	N	{5} + Dom(4) = {0,1,3,4,5}
6	N	{6} + Dom(4) = {0,1,3,4,6}
7	N	{7} + (Dom(5) ∩ Dom(6) ∩ Dom(10)) = {0,1,3,4,7}
8	N	{8} + Dom(7) = {0,1,3,4,7,8}
9	N	{9} + Dom(8) = {0,1,3,4,7,8,9}
10	N	{10} + Dom(8) = {0,1,3,4,7,8,10}

Dominator Example



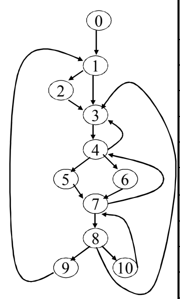
	Dom		
Block	initial	iteration1	iteration2
0	{0}	{0}	{0}
1	N	{0,1}	{0,1}
2	N	{0,1,2}	{0,1,2}
3	N	{0,1,3}	{0,1,3}
4	N	{0,1,3,4}	{0,1,3,4}
5	N	{0,1,3,4,5}	{0,1,3,4,5}
6	N	{0,1,3,4,6}	{0,1,3,4,6}
7	N	{0,1,3,4,7}	{0,1,3,4,7}
8	N	{0,1,3,4,7,8}	{0,1,3,4,7,8}
9	N	{0,1,3,4,7,8,9}	{0,1,3,4,7,8,9}
10	N	{0,1,3,4,7,8,10}	{0,1,3,4,7,8,10}

Computing IDOM from DOM

1. For each node *n*, initially set IDOM(*n*) = DOM(*n*)-{*n*} (SDOM - strict dominators)
2. For each node p in IDOM(n), see if p has dominators other than itself also included in IDOM(n): if so, remove them from IDOM(n)

- The immediate dominator *m* of *n* is the strict dominator of *n* that is closest to *n*

I-Dominator Example

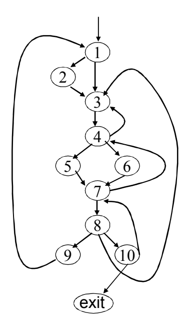


Block	IDom	
	initial (SDOM)	
0	{}	{}
1	{0}	{0}
2	{0,1}	{1} //0 - 1's dominator
3	{0,1}	{1} //0 - 1's dominator
4	{0,1,3}	{3} // 0,1 - 3's dominators
5	{0,1,3,4}	{4} // 0,1,3 - 4's dominators
6	{0,1,3,4}	{4} // 0,1,3 - 4's dominators
7	{0,1,3,4}	{4} // 0,1,3 - 4's dominators
8	{0,1,3,4,7}	{7} // 0,1,3,4 - 7's dominators
9	{0,1,3,4,7,8}	{8} // 0,1,3,4,7 - 8's dominators
10	{0,1,3,4,7,8}	{8} // 0,1,3,4,7 - 8's dominators

Post-Dominance

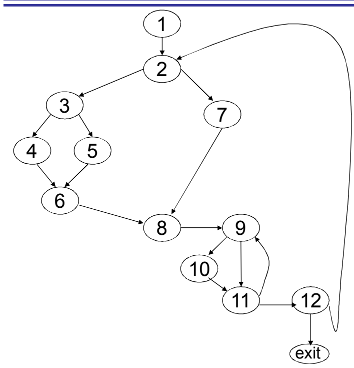
- Related concept
- Node *d* of a CFG *post-dominates* node *n* if every path from *n* to the exit node passes through *d* (*d pdom n*)
  - Pdom(*n*): the set of post-dominators of node *n*
  - Every node post-dominates itself: *n* ∈ Pdom(*n*)
- Each node *n* has a unique *immediate post dominator m*

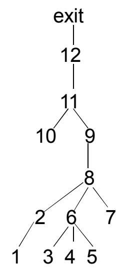
Post-dominator Example



Block	Pdom	IPdom
1	{3,4,7,8,10,exit}	3
2	{2,3,4,7,8,10,exit}	3
3	{3,4,7,8,10,exit}	4
4	{4,7,8,10,exit}	7
5	{5,7,8,10,exit}	7
6	{6,7,8,10,exit}	7
7	{7,8,10,exit}	8
8	{8,10,exit}	10
9	{1,3,4,7,8,10,exit}	1
10	{10,exit}	exit

CFG

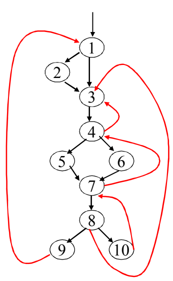




Natural Loops

- Natural loops that are suitable for improvement have two essential properties:
  - A loop must have a single entry point called *header*
  - There must be at least one way to iterate the loop, i.e., at least one path back to the header
- Identifying natural loops
  - Searching for ***back edges*** (*n*→*d*) in CFG whose heads *dominate* their tails
    - For an edge *a*→*b*, *b* is the *head* and *a* is the *tail*
    - A back edge flows from a node *n* to one of *n*'s dominators *d*
  - The natural loop for that edge is {*d*}+the set of nodes that can reach *n* without going through *d*
    - *d* is the header of the loop

Back Edge Example



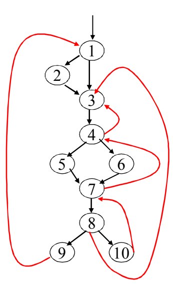
Back edges?

Block	Dom	IDom
1	1	—
2	1,2	1
3	1,3	1
4	1,3,4	3
5	1,3,4,5	4
6	1,3,4,6	4
7	1,3,4,7	4
8	1,3,4,7,8	7
9	1,3,4,7,8,9	8
10	1,3,4,7,8,10	8

Identifying Natural Loops

- Given a back edge *n*→*d*, the natural loop of the edge includes
  - Node *d*
  - Any node that can reach *n* without going through *d*
- Loop construction
  - Set *loop*={*d*}
  - Add *n* into *loop* if n ≠*d*
  - Consider each node *m*≠*d* that we know is in *loop*, make sure that *m*'s predecessors are also inserted in *loop*

Natural Loops Example

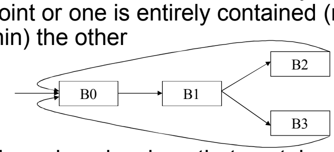


Back edge	Natural loop
10→7	{7,10,8}
7→4	{4,7,5,6 10,8}
4→3	{3,4,7,5,6,10,8}
8→3	{3,4,7,5,6,10,8}
9→1	{1,9,8,7,5,6, 10,4,3,2}

- Why neither {3,4} nor {4,5,6,7} is a natural loop?

Inner Loops

- A useful property of natural loops: unless two loops have the same header, they are either disjoint or one is entirely contained (nested within) the other

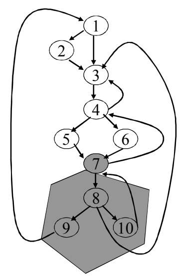


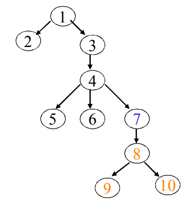
- An inner loop is a loop that contains no other loops
  - Good optimization candidate
  - The inner loop of the previous example: {7,8,10}

Dominance Frontiers

- For a node *n* in CFG, DF(*n*) denotes the dominance frontier set of *n*
  - DF(*n*) contains all nodes *x* s.t. *n* dominates an immediate predecessor of *x* but does not strictly dominate *x*
  - For this to happen, there is some path from node *n* to *x*, *n* → … → *y* → *x* where (*n* DOM *y)* but !(*n* SDOM *x)*
  - Informally, DF(*n*) contains the first nodes reachable from *n* that *n* does not strictly dominate, on each CFG path leaving *n*
- Used in SSA calculation and redundancy elimination

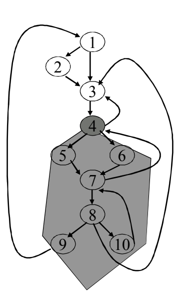
Dominance Frontier for Node 7

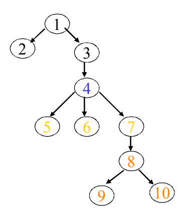




Paths of interest:
7 → 4
7 → 8 → 3
7 → 8 → 9 → 1
7 → 8 → 10 → 7
DF(7)={1,3,4,7}

Dominance Frontier for Node 4





Paths of interest:
DF(4)={1,3,4}

Computing Dominance Frontiers

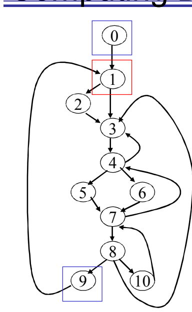
- Easiest way:
  $DF(x) = SUCC(DOM^{-1}(x)) - SDOM^{-1}(x)$ where SUCC(x) = set of successors of x in the CFG
  - But not the most efficient
- Observation
  - Nodes in a DF must be join nodes
  - The predecessor of any join node *j* must have *j* in its DF unless it dominates *j*
  - The dominators of *j*'s predecessors must have *j* in their DF sets unless they also dominate *j*

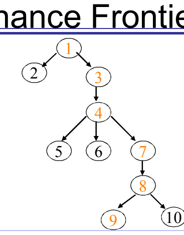
Computing Dominance Frontiers

```
for all nodes n, initialize DF(n) =Ø
for all nodes n
   if n has multiple predecessors, then
      for each predecessor p of n
         runner = p
         while (runner ≠IDom(n))
                 DF(runner) = DF(runner) ∪ {n}
                 runner = IDom(runner)
```

- First identify join nodes *j* in CFG
- Starting with *j*'s predecessors, walk up the dominator tree until we reach the immediate dominator of *j*
  - Node *j* should be included in the DF set of all the nodes we pass by except for *j*'s immediate dominator

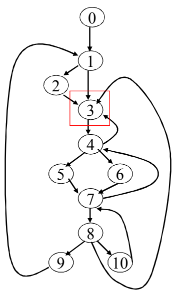
Computing Dominance Frontier

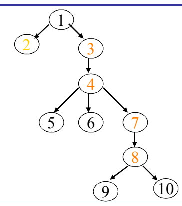




Join node 1:
runner = 0 = IDom(1)
runner = 9 : DF(9) += {1}
runner = 8 : DF(8) += {1}
runner = 7 : DF(7) += {1}
runner = 4 : DF(4) += {1}
runner = 3 : DF(3) += {1}
runner = 1 : DF(1) += {1}
runner = 0 = IDom(1)

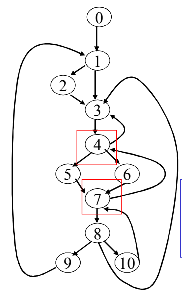
Computing Dominance Frontier

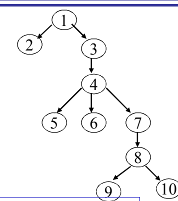




Join node 3:
runner = 1 = IDom(3)
runner = 2: DF(2) += {3}
runner = 4: DF(4) += {3}
runner = 3: DF(3) += {3}
runner = 8 : DF(8) += {3}
runner = 7 : DF(7) += {3}

Computing Dominance Frontier

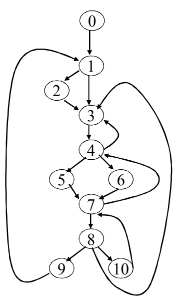




Join node 4:
runner = 3 = IDom(4)
runner = 7: DF(7) += {4}
runner = 4: DF(4) += {4}

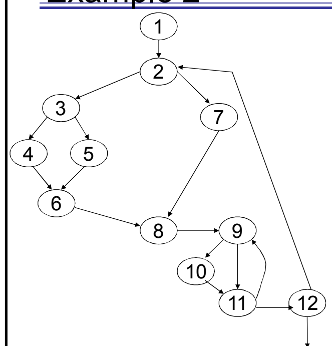
Join node 7:
runner = 5: DF(5) += {7}
runner = 6: DF(6) += {7}
runner = 10: DF(10) += {7}
runner = 8: DF(8) += {7}
runner = 7: DF(7) += {7}

Dominance Frontier Example



Block	DF
1	{1}
2	{3}
3	{1,3}
4	{1,3,4}
5	{7}
6	{7}
7	{1,3,4,7}
8	{1,3,7}
9	{1}
10	{7}

Example 2



Bloc k	DF
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	

Dominator-based Analysis

- Idea
  - Use dominators to discover loops for optimization
- Advantages
  - Sufficient for use by iterative data-flow analysis and optimizations
  - Least time-intensive to implement
  - Favored by most current optimizing compilers
- Alternative approach
  - Interval-based analysis/structural analysis

Summary

- CFG construction
  - Basic blocks identification
- CFG traversal
  - Depth-first spanning tree
  - Vertex ordering
- CFG analysis
  - Important regions: EBB and loop
  - Dominators
  - Dominance frontiers
- Additional references
  - Advanced compiler design and implementation, by S. Muchinick, Morgan Kaufmann